Multiband Parameter Estimation for Spectrum Sensing from Noisy Measurements

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***Abstract*—Under a dynamic spectrum access paradigm, a set of L spectrum bands licensed to primary users provide opportunities for an unlicensed secondary user to gain access to spectrum left idle by a primary user. We model the received noisy signal measurements on each band as a continuous-time Markov chain observed through a discrete-time Gaussian channel. Based on this model, we develop a scheme for estimating the parameters of the subset of $L^* < L$ bands that offer the "best" opportunities for dynamic spectrum access in the sense of largest mean idle periods. Our approach consists of a Markov modulated Gaussian process model, an associated expectation-maximization algorithm, and a computing budget allocation scheme for allocating sensing effort across the spectrum bands over a sequence of observation intervals. The sensing effort allocation scheme maximizes the probability that the L^* best bands will be determined from their parameter estimates obtained in the next observation interval. Simulation results are presented to demonstrate the performance of the proposed scheme.**[1]

***Index Terms*—Dynamic spectrum access, cognitive radio, spectrum sensing, expectation-maximization, simulation optimization**

I. Introduction

As the demand for wireless bandwidth increases, more efficient ways for utilizing the available spectrum must be developed. Spectrum measurement studies have shown that licensed spectrum bands allocated to primary users (PUs) can be highly underutilized. Dynamic spectrum access aims at increasing the utilization of the spectrum by allowing secondary users (SUs) to utilize idle channels in time, frequency or space without causing disruptions to the licensed PUs [1], [2]. To avoid interference to the PUs, spectrum sensing is required to determine when the spectrum bands are idle.

Spectrum sensing of a given band relies on inferring the active/idle state of the PU of the band from noisy received signal measurements. Prior work has shown that a model-based approach can improve the fidelity of spectrum sensing and also enables predictive sensing of the channel state. For example, a hidden Markov model (HMM) for spectrum sensing of a narrowband channel and an algorithm for estimating its parameter was proposed Nguyen *et al.* [3]. Deep learning approaches to spectrum sensing rely on fewer model assumptions, but require significantly more offline training [4].

[1]This work was supported in part by the U.S. National Science Foundation under Grants CNS-1205453 and CNS-1421869.
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In [2], a scheme was developed that essentially transforms the problem of joint wideband-temporal spectrum sensing into a multiband spectrum sensing problem. Multiband spectrum sensing scenarios arise naturally in other contexts as well. The computational effort required to perform spectrum sensing over a large set of spectrum bands can be prohibitive. In such scenarios, it would be desirable to allocate most of the sensing effort to a smaller subset of bands which provide the best opportunities for dynamic spectrum sensing.

In this paper, we focus on the problem of parameter estimation for L spectrum bands observed over a noisy channel. This problem involves determining, with high probability and with the smallest number of measurement samples, the subset of $L^* < L$ bands that provide the best spectrum access opportunities. We assume that the bands that have the largest mean PU idle periods are considered the best for dynamic spectrum access. In practice, typically $L^* \ll L$ and a given SU may set $L^* = 1$ to determine the single best band for dynamic spectrum access. Often, however, it is useful to have several alternative choices of bands when there are multiple SUs sharing the spectrum.

Our approach to the above problem is based on characterizing the received noisy measurements on a given band as the output of a continuous-time Markov chain passed through a discrete-time Gaussian channel. The resulting channel model is referred to as a Markov modulated Gaussian Process (MMGP), which was studied earlier by Roberts and Ephraim [5]. The MMGP is similar to the HMM applied to spectrum sensing in [3], except that the underlying process is a continuous-time rather than a discrete-time process. A key advantage of the continous-time chain is that the dynamics of the PU state process are decoupled from the measurement sampling.

We adopt the EM (expectation-maximization) algorithm in [5] to estimate the MMGP parameter of each frequency band in the multiband scenario. The EM algorithm is applied in conjunction with a scheme for allocating sensing time intervals among the frequency bands during a given observation interval. A sensing effort budget is allocated among the bands to determine, with high likelihood and in as few observation intervals as possible, a subset of the bands that provides the best spectrum access opportunities. Our proposed multiband parameter estimation scheme employs the so-called Optimal Computing Budget Allocation (OCBA) methodology of Chen (cf. [6]), which was originally developed in the context of simulation optimization. In our prior work [7], we applied OCBA to multiband parameter estimation in a simpler scenario in which channel noise is not present. The main contribution of the present work is an OCBA scheme for multiband parameter estimation with *noisy* measurements. We also extend the multiband parameter estimation scheme of [8], which aims to allocate sensing effort to minimize the total mean square estimation error over all bands by using the Cramér-Rao bound (CRB) in a noiseless scenario. For the noisy scenario, we resort to the complete data CRB for MMGP parameter estimation as a proxy for minimizing mean square error.

The remainder of the paper is organized as follows. In Section II, we state and set up the problem to be addressed. In Section III, we present the MMGP model for the noisy channel and discuss estimation of its parameter via the EM algorithm. In Section IV, we discuss two approaches for estimating the parameter estimation error, which are subsequently used in Section V to develop sensing time allocation schemes. Simulation results are presented in Section VI. Concluding remarks are provided in Section VII.

II. Problem Statement and Setup

We consider a scenario with L spectrum bands, each associated with a PU, which we identify with the set $\mathcal{L} := \{1, \ldots, L\}$. For simplicity, the bandwidths occupied by all spectrum bands are assumed to be equal. The associated PU on a given band alternates between active periods when it is transmitting data and idle periods during which it does not access the band. We denote the access process of the lth PU by $X(l) = \{X_t(l) : t \geq 0\}$, with state space $\mathcal{X} := \{0, 1\}$ such that $X_t(l) = 0$ when PU l is idle at time t and $X_t(l) = 1$ when the PU is transmitting at time t. The processes $\{X(l)\}_{l \in \mathcal{L}}$ are assumed to be statistically independent.

The process $X(l)$ can be modeled as a continuous-time Markov chain with generator matrix $Q(l) = \{q_{ij}(l), i, j \in \mathcal{X}\}$, where $q_{ij}(l)$ is the rate of the transition from state i to state j when $i \neq j$. We are primarily interested in estimating the generators $Q(l)$, since the initial distribution of a Markov chain, though technically a component of its parameter, has negligible long-term effects on the likelihood function. In a scenario without noise, suppose that k independent realizations of the access process of a given PU, $X = \{X_t : 0 \leq t \leq T\}$, are observed directly, where $T > 0$ is the observation time.[2] Let $M_{ij}^{(k)}(T)$ denote the number of jumps of X from state i to state j observed over the k trials and let $D_i^{(k)}(T)$ denote the total length of time in state i during the k trials. The maximum likelihood estimates (MLEs) of q_{ij} are given by [9, Sec. 4]:

$$\hat{q}_{ij}^{(k)} = \frac{M_{ij}^{(k)}(T)}{D_i^{(k)}(T)} \text{ if } i \neq j \text{ and } D_i^{(k)}(T) \neq 0. \qquad (1)$$

In the multiband case with L spectrum bands, the observation interval is divided into subintervals of lengths $T_1, \ldots, T_L$ such that $\sum_{l=1}^{L} T_l = T$. During the interval of length T, process $X(l)$ is observed only over the subinterval of length T_l. A key issue is how to allocate the durations $\{T_l\}$ of the sensing time subintervals.

In an online setup, Tehrani *et al.* [8] considered the problem of multiband parameter estimation, i.e., estimation of the parameters of $\{X(l)\}_{l \in \mathcal{L}}$, over a *sequence* of observation intervals of length T. In their scheme, at the end of the kth observation interval $O^{(k)}$, estimates $\{\hat{q}_{ij}^{(k)}(l)\}$ of the form (1) are computed for $k = 1, 2, \ldots$. The sensing time allocations $\{T_l^{(k+1)}\}$ for the next observation interval $O^{(k+1)}$ are computed with the objective of minimizing the total mean square error (MSE) with respect to the estimates $\{\hat{q}_{ij}^{(k+1)}(l)\}$. As a proxy for minimizing the MSE, a closed-form expression involving the CRB is minimized. We refer to this approach as CRB-based allocation or `CRBA`.

Rather than minimize the total MSE over all bands in $\mathcal{L}$, an alternative approach is to compute $\{T_l^{(k+1)}\}$ with the objective of maximizing the probability that the "best" subset of a given size, say $L^* \ll L$, among the bands is selected at the end of $O^{(k+1)}$. In this context "best" refers to the bands that provide the best dynamic spectrum access opportunities. In particular, the spectrum bands can be ranked in *decreasing* order of the values of the mean PU idle times, $[q_{01}(l)]^{-1}$. Our objective is to maximize the probability that the top L^* out of L bands are selected from ranking the estimates $[\hat{q}_{01}^{(k+1)}(l)]^{-1}$, $l \in \mathcal{L}$. To achieve this objective, we adopt the OCBA (Optimal Computing Budget Allocation) methodology from the field of simulation optimization [6].

In the present paper, we extend the `OCBA` scheme proposed in [7] by accommodating noisy measurements using a Markov modulated Gaussian Process (MMGP) model (see Section III). In this case, the underlying PU access processes $X(l)$ cannot be observed directly and parameter estimates of the form (1) are not applicable. We extend the `CRBA` scheme of [8] to take into account noisy measurements. We also propose a hybrid scheme `OCBA+CRBA`, which aims to select the best subset of bands, while minimizing the MSE of their parameter estimates.

III. Noisy Channel Model

We motivate the MMGP model for spectrum sensing with noisy measurements and discuss online parameter estimation based on the EM algorithm.

A. MMGP Model

The spectrum band over which the PU is observed is assumed to be flat Rayleigh fading with parameter σ_s combined with zero mean AWGN, defined by the circularly symmetric complex normal distribution $\mathcal{C}(0, \sigma_n^2)$. The mean SNR of the received PU signal is $\overline{\text{SNR}} = \sigma_s^2/\sigma_n^2$. Assume that the noisy PU signal is sampled uniformly by the SU in time at the sample rate f_s and the sample period $h = 1/f_s$. The observable process $Y = \{Y_k\}$ represents the power of

[2] We drop the index l when the identity of the PU is either obvious or immaterial to the discussion.

the faded noisy PU signal as received by the SU. The kth observation Y_k is defined by

$$Y_k = \frac{1}{W} \sum_{i=0}^{W-1} |Z_{kW+i}|^2, \tag{2}$$

where $Z = \{Z_k\}$ is the sampled process of the received PU signal and W is the number of samples for averaging. The samples $\{Z_k\}$ are conditionally i.i.d. Gaussian given by

$$Z_k \sim \begin{cases} \mathcal{C}(0, \sigma_n^2), & X_{t_k} = 0, \\ \mathcal{C}(0, \sigma_s^2 + \sigma_n^2), & X_{t_k} = 1. \end{cases} \tag{3}$$

where $t_k = kh$. The resulting energy estimates, Y_k, will be scaled chi-squared random variables with $2W$ degrees of freedom. Appealing to the central limit theorem, the probability law of Y_k can be approximated as conditionally Gaussian for sufficiently large W:

$$Y_k \sim \begin{cases} \mathcal{N}\left(2\sigma_n^2, \frac{4\sigma_n^4}{W}\right), & X_{t_k} = 0, \\ \mathcal{N}\left(2(\sigma_s^2 + \sigma_n^2), \frac{4(\sigma_s^2 + \sigma_n^2)^2}{W}\right), & X_{t_k} = 1, \end{cases} \tag{4}$$

where in this case $t_k = kWh$. Thus, we model Y as the output of a discrete-time memoryless Gaussian channel when the Markov chain X is applied as input.

The process (Y, X) is referred to as a Markov modulated Gaussian process (MMGP) and was studied by Roberts and Ephraim [5] in the context of ion-channel current estimation. In addition to the parameter of the underlying Markov chain, the MMGP parameter includes the mean values $\{\mu_i : i \in \mathcal{X}\}$ and the variances $\{\sigma_i^2 : i \in \mathcal{X}\}$ of the conditional Gaussian distribution. Thus, the parameter of interest for the MMGP can be expressed as $\phi = \big(Q, \{\mu_i\}, \{\sigma_i^2\}\big)$.

B. EM Algorithm

To efficiently estimate the parameter of the MMGP over time, we employ the EM algorithm developed in [5]. Over a finite interval of length T, the observations from a PU channel constitute a vector $Y^n = \{Y_k, k = 1, \ldots, n\}$, where $n = \lfloor \frac{T}{Wh} \rfloor$ and $\lfloor \cdot \rfloor$ denotes the floor operation. The likelihood function of MMGP parameter ϕ given the observable sequence Y^n in $[0, T]$ is denoted by $p(Y^n; \phi)$. Suppose $\phi = \big(Q, \{\mu_i\}, \{\sigma_i^2\}\big)$ is a current estimate value of the parameter of the MMGP (Y, X), and let $\hat{\phi} = \left(\hat{Q}, \{\hat{\mu}_i\}, \{\hat{\sigma}_i^2\}\right)$ denote the new parameter obtained at the end of an EM iteration.

Let $M_{ij}(t)$ denote, for $i \neq j$, the number of jumps of X from state i to state j in $[0, t]$. Define $\xi_k(i) := \mathbf{1}_{\{X_{t_k} = i\}}$, such that $\xi_k(i) = 1$ when the Markov chain was in state i observed at time t_k and $\xi_k(i) = 0$ otherwise. Let $N_i(t)$ denote the number of observations of $X = i$ that occurred in the interval $[0, t]$. Let $D_i(t)$ denote the total sojourn time of the Markov chain X in state i in the interval $[0, t]$. The new parameter estimate after an EM iteration is given by

$$\begin{aligned} &\hat{q}_{ij} = \frac{\hat{M}_{ij}}{\hat{D}_i}, \ i \neq j; \\ &\hat{\mu}_i = \frac{1}{\hat{N}_i} \sum_{k=1}^{n} \hat{\xi}_k(i) Y_k; \ \hat{\sigma_i^2} = \frac{1}{\hat{N}_i} \sum_{k=1}^{n} \hat{\xi}_k(i)(Y_k - \hat{\mu}_i)^2; \end{aligned} \tag{5}$$

where $\hat{M}_{ij}$, $\hat{N}_i$, $\hat{\xi}_k(i)$ and $\hat{D}_i$ are, respectively, conditional mean estimates under ϕ of $M_{ij}(T)$, $N_i(T)$, $\xi_k(i)$ and $D_i(T)$ given Y^n and ϕ. These intuitive expressions constitute the M-step of the EM algorithm. Explicit expressions for efficient evaluation of $\hat{M}_{ij}$, $\hat{D}_i$, $\hat{\xi}_t(i)$, and $\hat{N}_i$, using forward-backward recursions, are given in [5, Eqs. (27), (33), (35), (9)].

C. Recursive Parameter Estimation

A recursive parameter estimation algorithm is obtained by smoothing the parameter estimates obtained using the EM algorithm at the end of each observation interval. In particular, we employ a block smoothing approach in which the parameter estimate $\hat{\phi}(m+1)$ is weighted by the sensing times $\{T_l^{(k)} : k = 1, 2, \ldots, m+1\}$ allocated to the lth spectrum band as follows:

$$\tilde{\phi}(m+1) = \frac{\sum_{k=1}^{m} T_l^{(k)}}{\sum_{k=1}^{m+1} T_l^{(k)}} \tilde{\phi}(m) + \frac{T_l^{(m+1)}}{\sum_{k=1}^{m+1} T_l^{(k)}} \hat{\phi}(m+1). \tag{6}$$

The performance of this type of recursive estimation procedure applied to the EM algorithm was studied in [10].

IV. Parameter Estimation Error Estimates

We derive the complete data CRB, which provides a lower bound on the CRB for the MMGP. We also describe a numerical procedure for estimating the standard error for the EM algorithm.

A. Complete Data CRB

Let $p(Y^n; \phi)$ denote a parametric density or likelihood function of a sequence of random variables Y^n, and let $\phi = \{\phi_k, k = 1, \ldots, K\}$ denote the K-value parameter of that density. Then, the Fisher Information Matrix (FIM) $I_Y = \{I_Y(i, j), i, j = 1, \ldots, K\}$ is given by

$$I_Y(i, j) = -\mathsf{E}\left[\frac{\partial^2}{\partial \phi_i \partial \phi_j} \log p(Y^n; \phi)\right]. \tag{7}$$

Here we derive instead the *complete data* FIM $I_{YX} = \{I_{YX}(i, j), i, j = 1, \ldots, K\}$ where

$$I_{YX}(i, j) = -\mathsf{E}\left[\frac{\partial^2}{\partial \phi_i \partial \phi_j} \log p\left(\{X_t : 0 \leq t \leq T\}, Y^n; \phi\right)\right], \tag{8}$$

and $\{X_t : 0 \leq t \leq T\}$ denotes the underlying continuous-time Markov process in the observation interval $[0, T]$, and Y^n is the discrete-time observation sequence. It is easy to see that $I_Y^{-1} \geq I_{YX}^{-1}$ where the inequality is to be interpreted as $a'(I_Y^{-1} - I_{YX}^{-1})a \geq 0$ for any vector a of suitable dimension, and a' denotes the transpose of a.

The parameter of interest is given by

$$\phi = \{q_{01}, q_{10}, \mu_0, \mu_1, \sigma_0^2, \sigma_1^2\}. \tag{9}$$

From the log likelihood function of an MMGP (Y, X) in the interval $[0, T]$ given by [5, Eq. (5)] we obtain

$$I_{YX} = \text{diag}\left[\left\{\frac{\mathsf{E}[M_{ij}(T)]}{q_{ij}^2} : i, j \in \mathcal{X}, i \neq j\right\}, \left\{\frac{\mathsf{E}[N_i(T)]}{\sigma_i^2} : i \in \mathcal{X}\right\}, \left\{\frac{\mathsf{E}[N_i(T)]}{2\sigma_i^4} : i \in \mathcal{X}\right\}\right], \tag{10}$$

where we assume lexicographic ordering of the matrix elements along the diagonal. A closed-form expression for I_{YX} can be obtained using the following result.[3]

Proposition 1. *Consider a two-state MMGP observed in the interval $[0, t]$. Let $a \in \mathcal{X}$ and let $\bar{a} = 1 - a$. The expected number of jumps from state a to state $\bar{a}$ is given by*

$$\mathsf{E}[M_{a\bar{a}}(t)] = \frac{q_{01}q_{10}}{q}\left[t - \frac{1}{q}\left(1 - e^{-qt}\right)\right] + \frac{\pi_a q_{a\bar{a}}}{q}\left(1 - e^{-qt}\right), \tag{11}$$

where $q = q_{01} + q_{10}$. The expected total number of samples is given by

$$\mathsf{E}[N_a(t)] = \frac{q_{\bar{a}a}}{hq}t + \frac{\pi_a q_{a\bar{a}} - \pi_{\bar{a}} q_{\bar{a}a}}{hq^2}\left(1 - e^{-qt}\right). \tag{12}$$

A lower bound on the total MSE of the multiband parameter estimates is then given by

$$\sum_i I_Y^{-1}(i,i) \geq \sum_i I_{YX}^{-1}(i,i) = \sum_{l=1}^{L}\sum_{a\in\mathcal{X}}\left[\frac{q_{a\bar{a}}^2(l)}{\mathsf{E}[M_{a\bar{a}}(t,l)]} + \frac{\sigma_a^2(l) + 2\sigma_a^4(l)}{\mathsf{E}[N_a(t,l)]}\right], \tag{13}$$

where $M_{a\bar{a}}(t, l)$ denotes the number of jumps of $X(l)$ from state a to $\bar{a}$ and $N_a(t, l)$ denotes the time in state a in $[0, t]$.

B. EM Standard Error Estimation

We adopt the RES (Richardson Extrapolation Score vector) method [11] to estimate the standard error of the EM algorithm in terms of an estimate of the observed information matrix. The RES method estimates the Hessian $H(\phi)$ of the log likelihood function of ϕ by numerically differentiating the Fisher score vector $S(\phi)$, which is the gradient of the log likelihood function of ϕ, at the estimate $\hat{\phi}$ using the first-order Richardson extrapolation of the central difference. Then, the estimate $-\hat{H}(\phi)$ is the observed information matrix and $-\hat{H}^{-1}(\phi)$ is the estimated covariance matrix of ϕ. For a given PU access process X, let

$$f(\phi) = \mathsf{E}[\log p\,(Y^n, \{X_t : 0 \leq t \leq T\}; \phi) \mid Y^n, \phi]. \tag{14}$$

The Fisher score vector is given by $S(\phi) = \frac{\partial f(\phi)}{\partial \phi}$.

For the OCBA scheme discussed in Section V-B, we are particularly interested in obtaining an estimate of the standard error of the EM-based estimate of the cost function $J = q_{01}$, which is the inverse of the mean idle time of X in the idle state 0. The second-order partial derivative of $f(\phi)$ with respect to q_{01} can be approximated by the second-order central difference:

$$\frac{\partial^2 f(\phi)}{\partial q_{01}^2} \approx \mathcal{D}_1(\Delta) = \frac{f(\phi + \Delta\mathbf{e}_{q_{01}}) - 2f(\phi) + f(\phi - \Delta\mathbf{e}_{q_{01}})}{\Delta^2}, \tag{15}$$

with a truncation error of $O(\Delta^2)$, where $\mathbf{e}_{q_{01}}$ denotes a unit vector with a 1 in the position corresponding to q_{01} in ϕ. The first-order Richardson extrapolation of the central difference can be derived as [12]

$$\mathcal{D}_2(\Delta) = \frac{1}{3}[4\mathcal{D}_1(\Delta) - \mathcal{D}_1(2\Delta)]. \tag{16}$$

Substituting (15) into (16), we have

$$\mathcal{D}_2(\Delta) = \frac{1}{12\Delta^2}[-f(\phi - 2\Delta\mathbf{e}_{q_{01}}) + 16f(\phi - \Delta\mathbf{e}_{q_{01}}) - 30f(\phi) + 16f(\phi + \Delta\mathbf{e}_{q_{01}}) - f(\phi + 2\Delta\mathbf{e}_{q_{01}})], \tag{17}$$

where $\Delta = \eta \max(q_{01}, 1)$, see [11, Eq. 4]. An empirical choice of η is 0.01. Thus, we obtain the approximation of the second-order derivative of the log likelihood by (17) with a smaller truncation error of $O(\Delta^4)$. The standard deviation of the cost function can be estimated as $\hat{s} \approx [-\mathcal{D}_2(\Delta)]^{-\frac{1}{2}}$.

V. Multiband Sensing Time Allocation Schemes

In this section, we present the `CRBA` and `OCBA` schemes for multiband sensing time allocation with noisy measurements. We then discuss a hybrid scheme, `OCBA+CRBA`, which achieves the benefits of both approaches.

A. CRB-Based Allocation (`CRBA`)

For an observation interval of sufficiently large length $t = T$, minimization of the right-hand side of (13) subject to $\sum_{l=1}^{L} T_l = T$ yields an approximate, closed-form solution:

$$T_l = \frac{\sqrt{\alpha_l}}{\sum_{m=1}^{L}\sqrt{\alpha_m}}T, \tag{18}$$

where

$$\alpha_l = q(l)\sum_{a\in\mathcal{X}}\frac{1}{q_{\bar{a}a}(l)}[q_{a\bar{a}}(l) + h(\sigma_a^2(l) + 2\sigma_a^4(l))]. \tag{19}$$

In practice, T is finite, so we apply a sequential quadratic programming method [13] to minimize (13) subject to $\sum_{l=1}^{L} T_l = T$ and use the approximate solution given by (18) and (19) as a starting point.

B. OCBA Allocation (`OCBA`)

Our goal is to determine the L^* bands with largest mean sojourn time in the idle state, where typically $L^* \ll L$. Equivalently, we seek the L^* channels with minimum cost, where the cost function for spectrum band $l \in \mathcal{L}$ is defined by

$$J_l = q_{01}(l). \tag{20}$$

The sensing effort for the initial observation interval $O^{(1)}$ is simply allocated according to the `EQUAL` scheme. At the end

[3]Proof is omitted due to space constraints.

of $O^{(k)}$, $k \geq 1$, the PU parameter is re-estimated using the EM algorithm described in Section III-B. We then apply the OCBA methodology in [6] to determine sensing time allocations for the next observation interval $O^{(k+1)}$.

Let $\hat{J}_l = \hat{q}_{01}(l)$, denote an estimate of the cost for band $l \in \mathcal{L}$ in (20) obtained using the EM algorithm of Section III-B over an observation interval of duration Σ_l. In accordance with the setup for OCBA, we shall assume that the $\hat{J}_l$'s are independent and normally distributed,

$$\hat{J}_l \sim \mathcal{N}\left(J_l, \frac{v_l}{\Sigma_l}\right), \; l \in \mathcal{L}, \tag{21}$$

where J_l is the true cost value and v_l is the variance of the estimate for band l obtained over a unit time interval. This approximation is motivated by the asymptotic normality of maximum likelihood estimates and the fact that the accuracy of the estimate increases as Σ_l increases.

Let $s^{(k)}(l) = \sqrt{v_l/\Sigma_l^{(k)}}$ denote the standard deviation of the cost function estimate for band l at the end of observation interval $O^{(k)}$. An estimate of $s^{(k)}(l)$, which we denote by $\hat{s}^{(k)}(l)$, is obtained using the method discussed in Section IV-B. Let $\hat{J}_{i_\ell}$ be the ℓ-th order statistic of $\{\hat{J}_1, \hat{J}_2, \ldots, \hat{J}_L\}$, i.e., $\hat{J}_{i_1} \leq \hat{J}_{i_2} \leq \ldots \leq \hat{J}_{i_L}$. The selected subset is given by $\mathcal{L}^* := \{i_1, i_2, \ldots, i_{L^*}\}$. Define a scalar $c \in [\hat{J}_{i_{L^*}}, \hat{J}_{i_{L^*+1}}]$ as follows [6]:

$$c := \frac{\hat{s}^{(k)}(i_{L^*+1})\hat{J}_{i_L} + \hat{s}^{(k)}(i_{L^*})\hat{J}_{i_{L^*+1}}}{\hat{s}^{(k)}(i_{L^*}) + \hat{s}^{(k)}(i_{L^*+1})}. \tag{22}$$

The sensing time intervals $\Sigma_l^{(k+1)}$, $l \in \mathcal{L}$, are computed as the unique solution of the following equations:

$$\frac{\Sigma_1^{(k+1)}}{\left(\hat{s}^{(k)}(1)/(\hat{J}_1 - c)\right)^2} = \ldots = \frac{\Sigma_L^{(k+1)}}{\left(\hat{s}^{(k)}(L)/(\hat{J}_L - c)\right)^2}, \tag{23}$$

$$\sum_{l=1}^{L}\left(\Sigma_l^{(k+1)} - \Sigma_l^{(k)}\right) = T. \tag{24}$$

The sensing time allocations for $O^{(k+1)}$ are then given by

$$T_l^{(k+1)} = \max\left\{\Sigma_l^{(k+1)} - \Sigma_l^{(k)}, 0\right\}; \; l \in \mathcal{L}. \tag{25}$$

C. Hybrid OCBA/CRBA (`OCBA+CRBA`)

The OCBA procedure described above can be repeated during each subsequent observation interval until some maximum sensing time is exceeded. Alternatively, the procedure can be repeated until, the probability of correct selection (PCS), i.e., the probability that the bandes in $\mathcal{L}^*$ are indeed among the top L^* with respect to minimum cost, reaches some threshold. Since the PCS is unknown, a lower bound approximation, referred to as APCS (approximate PCS) can be used, where [6, Eq. (5.19)]

$$\text{APCS} := \prod_{l \in \mathcal{L}^*} \mathsf{P}\{\hat{J}_l \leq c\} \cdot \prod_{l \notin \mathcal{L}^*} \mathsf{P}\{\hat{J}_l \geq c\}, \tag{26}$$

which can be easily computed via the normality assumption on the $\hat{J}_l$'s given in (21). Motivated by the goal of choosing the best subset with high likelihood while obtaining accurate parameter estimates, we propose a hybrid OCBA/CRBA algorithm (`OCBA+CRBA`) in which OCBA is applied to all L bands until APCS exceeds a threshold p^*. Then CRBA is applied to the chosen subset of L^* bands to further improve the accuracy of the associated parameter estimates in the subsequent observation intervals.

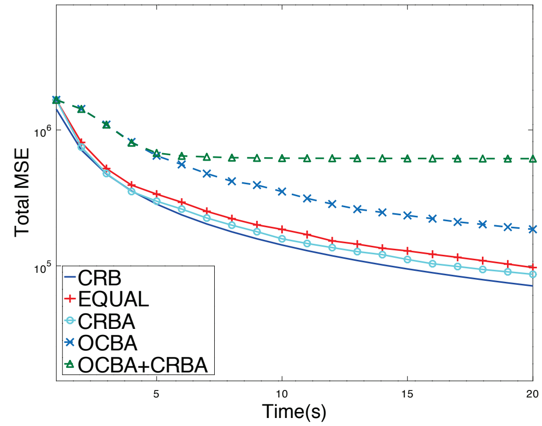

Fig. 1. Estimated total MSE for all channels.

VI. Numerical Results

We present simulation results to demonstrate and compare the performance of the proposed schemes.

A. Total MSE Over All and Best Bands

Figure 1 depicts the total MSE of the MMGP parameter estimates for all bands in a scenario with simulation parameters specified in Tables I and II. The observation interval for each iteration is $T = 1$ s. The `CRB` curve is computed using the right-hand side of (13). Observe that the `CRBA` and `EQUAL` curves both approach the `CRB` curve, whereas `OCBA` and `OCBA+CRBA` do not. This result is expected since OCBA concentrates the sensing effort on the best channels while allocating little or no sensing time to the remaining bands. The switchover between OCBA and CRBA in `OCBA+CRBA` occurs at about $t = 4$ on average. If we focus on only the best $L^* = 3$ channels, as shown in Fig. 2, the total MSE for `OCBA+CRBA` quickly becomes smaller than the other approaches. For this scenario, `EQUAL` performs slightly better than `CRBA`.

B. Probability of Correct Selection

To evaluate the performance of the various allocations schemes with respect to PCS, we executed $N_{\text{sim}} = 1000$ simulation runs of a scenario with $L = 25$ bands. In each simulation run, the values of the parameter components q_{01} and q_{10} were selected randomly according to a uniform distribution in the interval $[1250, 3750]$. In this experiment, $T = 2.5$ s. The other simulation parameters were the same as in Table II. For all the allocation schemes compared in this experiment, the $L^* = 3$ bands with the smallest estimates of q_{01} formed the selected subset. Figure 3 shows PCS of

	PU 1	PU 2	PU 3	PU 4	PU 5	PU 6	PU 7	PU 8	PU 9	PU 10
q_{01}	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
q_{10}	1000	3500	500	500	2000	4000	2000	4500	500	2000

TABLE I
MARKOV CHAIN PARAMETERS FOR EACH 10 BANDS.

Parameter	**Value**	**Description**
N_{sim}	200	number of simulations
N_{sense}	20	number of sensing iterations
W	5	number of samples averaged
L^*	3	number of "best" bands for `OCBA`
$T_l^{(1)}$	0.1 s	initial allocation for band l
SNR	10 dB	Signal-to-Noise Ratio
h	10^{-5} s	sampling interval
p^*	0.98	APCS threshold for `OCBA+CRBA`

TABLE II
SIMULATION PARAMETERS.

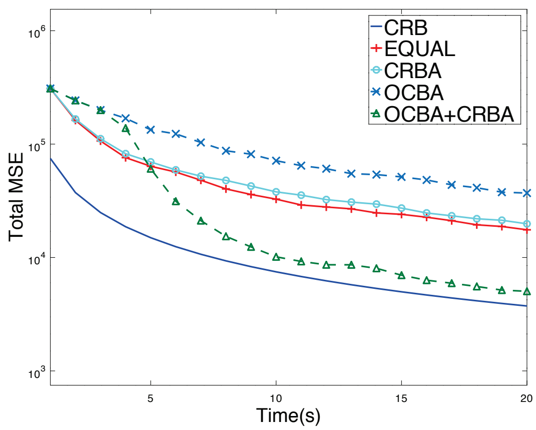

Fig. 2. Estimated total MSE for the best 3 channels.

the different estimation schemes vs. time. We see that `OCBA` and `OCBA+CRBA` have similar performance and both perform significantly better than `CRBA` and `EQUAL`. In effect, `OCBA` identifies the best channels quickly and allocates most of the sensing time to them, while `CRBA` and `EQUAL` "waste" more of the sensing time on other channels. For `OCBA+CRBA`, the switch to `CRBA` occurs at about $t = 25$ on average.

VII. CONCLUSION

We have applied the MMGP model to the problem of multiband spectrum sensing with noisy measurements and developed multiband parameter estimation schemes that allocate sensing effort among the bands. The proposed `OCBA+CRBA` scheme is designed with the goal of maximizing the probability of selecting a smaller subset of the best bands with respect to dynamic spectrum access opportunities while minimizing the mean square error of their parameter estimates. The computational advantage of the proposed scheme with respect to a simple equal allocation scheme increases with the number of spectrum bands under consideration.

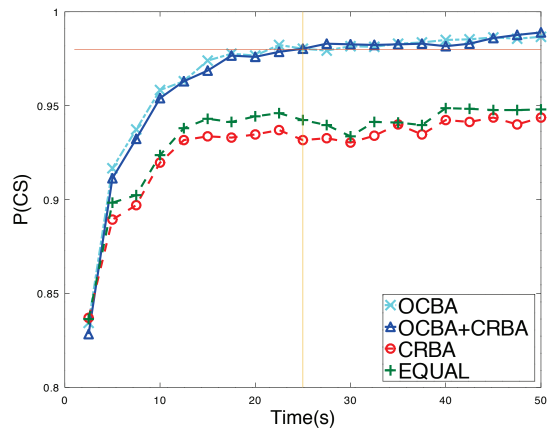

Fig. 3. Probability of correct selection.

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