Multiband Spectrum Sensing with Non-exponential Channel Occupancy Times

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***Abstract*—In a wireless network with dynamic spectrum sharing, tracking temporal spectrum holes across a wide spectrum band is a challenging task. We consider a scenario in which the spectrum is divided into a large number of bands or channels, each of which has the potential to provide dynamic spectrum access opportunities. The occupancy times of each band by primary users are generally non-exponentially distributed. We develop an approach to determine and parameterize a small selected subset of the bands with good spectrum access opportunities, using limited computational resources under noisy measurements. We model the noisy measurements of the received signal in each band as a bivariate Markov modulated Gaussian process, which can be viewed as a continuous-time bivariate Markov chain observed through Gaussian noise. The underlying bivariate Markov process allows for the characterization of non-exponentially distributed state sojourn times. The proposed scheme combines an online expectation-maximization algorithm for parameter estimation with a computing budget allocation algorithm. Observation time is allocated across the bands to determine the subset of G^* out of G frequency bands with the largest mean idle times for dynamic spectrum access and at the same time to obtain accurate parameter estimates for this subset of bands. Our simulation results show that when channel holding times are non-exponential, the proposed scheme achieves a substantial improvement in the probability of correct selection of the best subset of bands compared to an approach based on a (univariate) Markov modulated Gaussian process model.**

***Index Terms*—Dynamic spectrum access, cognitive radio, spectrum sensing, expectation-maximization, computing budget allocation, hidden Markov model, machine learning.**

I. Introduction

Various spectrum sensing algorithms have been developed to improve the accuracy and effectiveness of identifying spectrum holes in dynamic spectrum access networks. Spectrum sensing approaches such as energy detection, matched filter detection and cyclostationary feature detection are quite mature and have been widely investigated for narrowband and wideband spectrum sensing [1]. A wideband edge detector provides an efficient way to identify the idle spectral bands over a wide spectrum by detecting discontinuities in the power spectral density [2]. For a very wide spectrum band, various compressive sensing schemes have been proposed to recover and sense the spectrum from the sub-Nyquist sampled signal [3].

Temporal spectrum sensing of a narrowband channel based on a hidden Markov model (HMM) of primary user (PU) transmission activity was proposed in [4]. The proposed scheme involves estimation of the parameter of an HMM from energy measurements via the Baum algorithm. Given the parameter estimate, decisions on spectrum occupancy of the channel are made using the maximum-a-posteriori (MAP) decision rule. In the HMM, the sojourn times in the active and idle states are implicitly geometrically distributed. To relax this assumption, the scheme was extended in [5] using a hidden *bivariate* Markov model (HBMM), in which the state sojourn times have discrete phase-type distributions. In [6], an online parameter estimation scheme for HBMM-based spectrum sensing was developed to bypass offline training of the model and to adapt to changes in the PU transmission pattern.

In the wideband regime, HMM models were used in [7], [8] to transform wideband sensing into a multiband sensing problem. Narrowband temporal sensing techniques can then be applied to each of the identified primary user (PU) spectrum bands[1]. This approach has the advantage of avoiding the degradation in spectrum sensing accuracy that can arise in wideband sensing schemes that do not take into account temporal dynamics, due to the presence of low duty cycles in PU transmissions. On the other hand, if the number of identified PU bands is large, the application of narrowband sensing can be costly in terms of computational resources and sensing time.

To address these issues, we proposed a multiband sensing scheme [9] that combines parameter estimation and a computing budget allocation scheme to allocate observation time among the different bands with the aim of determining the best sensed subset of bands subject to a computing budget. A continuous-time Markov process was used to model the underlying temporal dynamics of the PU transmission. To take into account noisy measurements, the received signal from a PU is modeled by a Markov Modulated Gaussian Process (MMGP) [10]. A key advantage of the continuous-time model relative to the discrete-time HMM is that the temporal dynamics of the process are decoupled from the sampling rate of the observed signal. Similar to the HMM, however, in an MMGP, the underlying state sojourn times are exponentially distributed.

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[1]We use the terms band and channel interchangeably.

In this paper, we extend the multiband parameter estimation scheme introduced in [9] to accommodate *non-exponential* occupancy and idle times by modeling PU transmission on a given band as a *bivariate* Markov Markov Modulate Gaussian Process (BMMGP). The underlying state sojourn times of the BMMGP are phase-type distributed, which allows for a more accurate characterization of the PU transmission process on a given band compared to the MMGP used in [9].

To estimate the parameter of a BMMGP, we extend the expectation-maximization (EM) algorithm for the MMGP in [10]. In [9], the optimal computing budget allocation (OCBA) scheme of Chen [11] is used to determine the observation time allocations for estimating the MMGP parameters of a set of frequency bands. Due to the higher complexity of the BMMGP model, a different approach is needed to apply OCBA to the observation time allocation problem. Our simulation results show that more accurate multiband spectrum sensing can be achieved using the BMMGP model compared to the MMGP model when the channel occupancy times are non-exponential. The BMMGP-based multiband parameter estimation scheme achieves a significantly higher probability of correct selection of the best subset G^* of G bands, where typically $G^* \ll G$.

The remainder of the paper is organized as follows. In Section II, we present the BMMGP model for PU transmissions on a noisy channel and develop the EM algorithm for the BMMGP. In Section III, we adapt the OCBA scheme to perform observation time budget allocation under the BMMGP model. Simulation results are presented in Section IV. Concluding remarks are provided in Section V.

II. BMMGP Model and Parameter Estimation

In this section, we discuss the BMMGP and formulate an EM algorithm to estimate its parameter.

A. Bivariate Markov Modulated Gaussian Process

Assume the state process of a PU channel is a continuous-time process $X = \{X_t : t \geq 0\}$ with finite state space $\mathcal{X} := \{1, \ldots, d\}$, where d is the number of states of X. For the spectrum sensing application $d = 2$ and we let states 1 and 2 represent the idle and active states, respectively of the PU channel. We define a joint process $Z = (X, S)$ as a homogeneous continuous-time bivariate Markov chain [12]. The auxiliary process $S = \{S_t : t \geq 0\}$ has finite state space $\mathcal{S} := \{1, \ldots, r\}$, where r corresponds to the number of phases in the state sojourn time distribution of X, which belongs to the class of phase-type distributions. The state space of Z is given by $\mathcal{Z} = \mathcal{X} \times \mathcal{S}$ with dr states. By introducing the auxiliary process S, the state sojourn time of X becomes phase-type rather than exponential. Let $Y = \{Y_k : k = 0, 1, \ldots\}$ represent the discrete-time energy measurement process sampled uniformly with sampling interval h at the receiver of a SU. The time epoch corresponding to Y_k is given by $t_k = kh$. The process Y is conditionally Gaussian and independent of S given X. The trivariate process (Y, X, S) is referred to as a bivariate Markov modulated Gaussian process.

Let P denote the underlying probability measure and let $p(\cdot)$ denote an associated probability density function. The parameter of a BMMGP is given by

$$\phi = (Q, \{\mu_a : a \in \mathcal{X}\}, \{\sigma_a^2 : a \in \mathcal{X}\}), \tag{1}$$

where $Q = \{q_{ab}(ij) : a, b \in \mathcal{X}, i, j \in \mathcal{S}\}$ denotes the generator matrix of the bivariate Markov chain, $\{\mu_a : a \in \mathcal{X}\}$ and $\{\sigma_a^2 : a \in \mathcal{X}\}$ are the mean and variance of the conditional Gaussian density $p_{Y_k}(y \mid X_{t_k} = a)$ for all k and $a \in \mathcal{X}$. Assume the bivariate states $\{(a, i) \in \mathcal{Z}\}$ are ordered lexicographically. The generator matrix Q is a $dr \times dr$ matrix with off-diagonal elements $\{q_{ab}(ij) : (a, i) \neq (b, j)\}$, where $q_{ab}(ij)$ is the transition rate from state (a, i) to (b, j), and with diagonal elements given by

$$q_{aa}(ii) = -\sum_{(b,j) \neq (a,i)} q_{ab}(ij), \; i = 1, \ldots, r. \tag{2}$$

Let the generator matrix Q be partitioned into some submatrices such that $Q = \{Q_{ab} : a, b \in \mathcal{X}\}$, where $Q_{ab} = \{q_{ab}(ij) : i, j \in \mathcal{S}\}$. Assume that the submatrix Q_{aa} is irreducible, such that Q_{aa} is nonsingular and all elements of the matrices $-Q_{aa}^{-1}$ and $e^{Q_{aa}t}$ are positive for all $t > 0$.

Let $F(y) = \{F_{ij}^{ab}(y) : a, b \in \mathcal{X}, i, j \in \mathcal{S}\}$ denote the transition probability matrix of the BMMGP, where

$$\begin{aligned} F_{ij}^{ab}(y) &= \mathsf{P}\left(Y_k \leq y, Z_{t_k} = (b,j) \mid Z_{t_{k-1}} = (a,i)\right) \\ &= \mathsf{P}\left(Y_k \leq y \mid Z_{t_k} = (b,j), Z_{t_{k-1}} = (a,i)\right) \cdot \\ &\qquad \mathsf{P}\left(Z_{t_k} = (b,j) \mid Z_{t_{k-1}} = (a,i)\right) \\ &= \mathsf{P}\left(Y_k \leq y \mid X_{t_k} = b\right) \cdot \\ &\qquad \mathsf{P}\left(Z_{t_k} = (b,j) \mid Z_{t_{k-1}} = (a,i)\right). \end{aligned} \tag{3}$$

Note that the density of Y_k given state X_{t_k} is independent of S. The transition probability from $Z_{t_{k-1}}$ to Z_{t_k} is given by

$$\mathsf{P}\left(Z_{t_k} = (b,j) \mid Z_{t_{k-1}} = (a,i)\right) = \left[e^{Qh}\right]_{ij}^{ab}, \tag{4}$$

where $[\cdot]_{ij}^{ab}$ denotes the (i, j) element in the (a, b) submatrix. Taking the derivative of $F_{ij}^{ab}(y)$ with respect to y, the elements of the transition density matrix $f(y)$ are given by

$$f_{ij}^{ab}(y) = p_{Y_k}\left(y \mid X_{t_k} = b\right) \left[e^{Qh}\right]_{ij}^{ab}. \tag{5}$$

Let $f^{ab}(y)$ denote the (a, b) submatrix such that $f(y) = \{f^{ab}(y) : a, b \in \mathcal{X}\}$. Define an $dr \times dr$ block diagonal matrix B_k with diagonal blocks given by $\{p_{Y_k}\left(y \mid X_{t_k} = a\right) I : a \in \mathcal{X}\}$, where I is an $r \times r$ identity matrix. Then, the transition density matrix can be expressed as

$$f(y) = e^{Qh} B_k. \tag{6}$$

Supposed that the underlying continuous-time bivariate Markov chain Z is sampled at its jump points to obtain a sampled bivariate Markov chain $\tilde{Z}$. Denote the transition matrix of $\tilde{Z}$ by $D = \{D_{ab} : a, b \in \mathcal{X}\}$, where D_{ab} is a $r \times r$ submatrix. The submatrix D_{ab} can be expressed as [13], [14]

$$D_{ab} = \begin{cases} -Q_{aa}^{-1} Q_{ab}, & a \neq b, \\ \mathbf{0}, & \text{otherwise}. \end{cases} \tag{7}$$

Let a dr row vector ν denote the stationary distribution of $\tilde{Z}$, expressed as $\nu = \{\nu_a : a \in \mathcal{X}\}$ where ν_a is a size r row vector of the conditional stationary distribution of state S given $X = a$. It satisfies

$$\nu D = \nu, \ \nu \mathbf{1} = 1. \tag{8}$$

With the stationary distribution of the sampled bivariate Markov chain, we are able to obtain the distribution of the state sojourn time X, which is discussed in Section III.

B. Forward-Backward Recursion

The forward-backward recursions for a BMMGP are similar to those given in [10] for an MMGP. Given a sequence $\{y_k\}$, we use the notations $y_k^n := \{y_k, y_{k+1}, \ldots, y_n\}$ when $k < n$ and $y^n := y_0^n = \{y_0, \ldots, y_n\}$. Let $\pi = \text{row}\{\pi_a : a \in \mathcal{X}\}$, be a dr row vector of the initial state probabilities of the bivariate Markov chain Z, where $\pi_a = \{\pi_{a,i} = \mathsf{P}(Z_0 = (a,i)) : i \in \mathcal{S}\}$. For $k = 0, 1, \ldots, n$, let $L(k)$ denote a dr row vector of the forward densities, i.e., $L(k) = \{L^a(k) : a \in \mathcal{X}\}$, where $L^a(k) = \{L_i^a(k) = p(Y^k, Z_{t_k} = (a,i)) : i \in \mathcal{S}\}$. The forward recursion is given by

$$\begin{aligned} L(0) &= \pi B_0, \\ L(k) &= L(k-1) f(Y_k), \ k = 1, \ldots, n. \end{aligned} \tag{9}$$

Let $R(k)$ denote a dr column vector of the backward densities, i.e., $R(k) = \text{col}\{R^a(k) : a \in \mathcal{X}\}$, where $R^a(k) = \text{col}\{R_i^a(k) = p(Y_{k+1}^n \mid Z_{t_k} = (a,i)) : i \in \mathcal{S}\}$. The backward recursion is given by

$$\begin{aligned} R(n) &= \mathbf{1}, \\ R(k) &= f(Y_k) R(k+1), \ k = n-1, \ldots, 0, \end{aligned} \tag{10}$$

where $\mathbf{1}$ denotes a column vector of all ones. The likelihood of the observation sequence is given by

$$\mathsf{P}(Y^n) = L(n)\mathbf{1} = \pi B_0 \prod_{k=1}^{n} f(Y_k)\mathbf{1}. \tag{11}$$

To guarantee the numerical stability of the forward and backward recursions, we need to scale the densities appropriately. The scaled forward recursion is given by

$$\begin{aligned} \bar{L}(0) &= \frac{\pi B_0}{c_0}, \\ \bar{L}(k) &= \frac{\bar{L}(k-1) f(Y_k)}{c_k}, \ k = 1, \ldots, n, \end{aligned} \tag{12}$$

where

$$\begin{aligned} c_0 &= p(Y_0) = \pi B_0 \mathbf{1}, \\ c_k &= p(Y_k \mid Y^{k-1}) = \bar{L}(k-1) f(Y_k)\mathbf{1}, \ k = 1, \ldots, n. \end{aligned} \tag{13}$$

The scaled backward recursion is given by

$$\begin{aligned} \bar{R}(n) &= \mathbf{1}, \\ \bar{R}(k) &= \frac{f(Y_k)\bar{R}(k+1)}{c_k}, \ k = n-1, \cdots, 0. \end{aligned} \tag{14}$$

The scaled and unscaled vectors are related by

$$\bar{L}(k) = \frac{L(k)}{\prod_{i=0}^{k} c_i}, \ \bar{R}(k) = \frac{R(k)}{\prod_{i=k}^{n} c_i}, \tag{15}$$

respectively. The scaled forward density can be expressed as

$$\bar{L}(k) = p(Z_{t_k} \mid Y^k), \tag{16}$$

for $k = 0, \ldots, n$. The likelihood in (11) can be expressed in terms of c_k as follows:

$$\mathsf{P}(Y^n) = L(n)\mathbf{1} = \left(\prod_{k=0}^{n} c_k \right) \bar{L}(n)\mathbf{1} = \prod_{k=0}^{n} c_k. \tag{17}$$

C. EM algorithm

Let $M_{ij}^{ab}(T)$ denote, for $(a,i) \neq (b,j)$, the number of jumps of Z from state (a,i) to state (b,j) in $[0,T]$. Let $D_i^a(T)$ denote the total sojourn time of the bivariate Markov chain Z in state (a,i) in the interval $[0,T]$. Define $\xi_k(a) := \mathbf{1}_{\{X_{t_k}=a\}}$, such that $\xi_k(a) = 1$ when the state X of the bivariate Markov chain is in state a at time t_k and $\xi_k(a) = 0$ otherwise. Let $N_a(T)$ denote the number of observations occurred in state a in the interval $[0,T]$. Let $n = \lfloor T/h \rfloor$ denote the index of the largest time epoch t_k in $[0,T]$. The new parameter estimate after an EM iteration is given by

$$\hat{q}_{ij}^{ab} = \frac{\hat{M}_{ij}^{ab}}{\hat{D}_i^a}, \ (a,i) \neq (b,j), \tag{18}$$

$$\hat{\mu}_a = \frac{1}{\hat{N}_a} \sum_{k=0}^{n} \hat{\xi}_k(a) Y_k, \tag{19}$$

$$\hat{\sigma}_a^2 = \frac{1}{\hat{N}_a} \sum_{k=0}^{n} \hat{\xi}_k(a)(Y_k - \hat{\mu}_a)^2, \tag{20}$$

where $\hat{M}_{ij}^{ab}$, $\hat{D}_i^a$, $\hat{N}_i$ and $\hat{\xi}_k(a)$ are, respectively, conditional mean estimates of $M_{ij}^{ab}(T)$, $D_i^a(T)$, $N_a(T)$ and $\xi_k(a)$ given Y^n and ϕ. These expressions constitute the M-step of the EM algorithm.

The new estimate of $M_{ij}^{ab}(T)$ in the mth EM iteration can be expressed as

$$\begin{aligned} \hat{M}_{ij}^{ab}(T) &= \mathsf{E}\left[M_{ij}^{ab}(T) \mid Y^n; \phi_m \right] \\ &= \sum_{t \in [0,T]} \mathsf{P}\left(Z_{t-} = (a,i), Z_t = (b,j) \mid Y^n; \phi_m \right). \end{aligned}$$

In [13], X was observable and the estimation was divided into two cases: $(a = b, i \neq j)$ and $(a \neq b)$. This is feasible for the parameter estimation of a continuous-time Markov chain, but not for a BMMGP, since the Markov chain is not directly observable. Instead, we use a similar approach to that in [10]. Define a $2dr \times 2dr$ block matrix

$$C = \begin{bmatrix} Q & \sum_{k=1}^{n} \frac{1}{c_k} B_k \bar{R}(k) \bar{L}(k-1) \\ \mathbf{0} & Q \end{bmatrix}, \tag{21}$$

where $\mathbf{0}$ is a $dr \times dr$ zero matrix. Define a $dr \times dr$ matrix

$$\mathcal{I} = \left[e^{Ch} \right]_{12}, \tag{22}$$

which is the upper right block matrix of the matrix exponential e^{Ch}. Then, the estimate $\hat{M}_{ij}^{ab}$ is given by

$$\hat{M}_{ij}^{ab} = \left[Q \odot \mathcal{I}'\right]_{ij}^{ab}, \tag{23}$$

where $'$ denotes matrix transpose and $\odot$ denotes element-wise matrix multiplication.

Next, the new estimate $\hat{D}_i^a(T)$ in the mth EM iteration is given by

$$\hat{D}_i^a(T) = \mathsf{E}\left[D_i^a(T) \mid Y^n; \phi_m\right] = \int_0^T \mathsf{P}\left(Z_t = (a,i) \mid Y^n\right) dt.$$

Similar to the derivation of $\hat{M}_{ij}^{ab}$, we obtain the estimate $\hat{D}_i^a(T) = \hat{M}_{ii}^{aa}/q_{ii}^{aa}$. Based on [10, Eq. (35)], the estimate of $\xi_k(a)$ is given by

$$\hat{\xi}_k(a) = \sum_{i\in\mathcal{S}} \mathsf{P}\left(Z_{t_k} = (a,i) \mid Y^n\right) = \sum_{i\in\mathcal{S}} \bar{L}_i^a(k)\bar{R}_i^a(k).$$

Then, the estimate of N_a is given by $\hat{N}_a = \sum_{k=0}^n \hat{\xi}_k(a)$. Obtaining the estimates $\hat{M}_{ij}^{ab}$, $\hat{D}_i^a$, $\hat{N}_i$ and $\hat{\xi}_k(a)$ constitutes the E-step of the EM algorithm.

D. Parameter Estimation with Block Smoothing

Since the multiband parameter estimation is performed over a sequence of observation intervals, improved estimation accuracy can be obtained by applying a block smoothing approach to the EM estimate at the end of each observation interval as in [9]. Consider a given PU channel modeled by a BMMGP. Assuming that the BMMGP is stationary, the smoothed estimate $\tilde{\phi}(m+1)$ for the gth PU band in the $(m+1)$th observation interval is given by

$$\tilde{\phi}(m+1) = \frac{\sum_{k=1}^m T^{(k)}}{\sum_{k=1}^{m+1} T^{(k)}}\tilde{\phi}(m) + \frac{T^{(m+1)}}{\sum_{k=1}^{m+1} T^{(k)}}\hat{\phi}(m+1),$$

where $T^{(k)}$ denotes the observation subinterval duration allocated to the PU channel during the kth observation period, and $\hat{\phi}(m+1)$ denotes the EM estimate in the $(m+1)$th observation period. Thus, the final parameter estimate for the current observation period is an average of the EM estimates from the beginning to the current observation period. The performance of block smoothing applied to the EM algorithm was studied in [15].

III. Multiband Time Budget Allocation

In this section, we present the multiband time budget allocation scheme for the PU channels modeled by BMMGPs with $d = 2$ using the OCBA methodology. In this context, the goal of OCBA is to determine the G^* out of G PU bands with the largest mean sojourn times in the idle state. Let $\mathcal{G} = \{1, \ldots, G\}$ denote the set of all PU channels. Let $d_a(g)$ denote the mean sojourn time of the gth band given $X = a$. Since OCBA seeks the optimal subset with minimum cost, the objective function for the PU band $g \in \mathcal{G}$ is defined by the negative mean idle time,

$$J_g = -d_1(g). \tag{24}$$

The objective functions, $\{J_g : g \in \mathcal{G}\}$, are assumed to be independent and normally distributed.

A. OCBA-based Observation Time Allocation

The general OCBA algorithm is described in [11]. Its application to multiband parameter estimation for spectrum sensing was first proposed in [16] and [9]. We extend the multiband parameter estimation approach by employing a BMGPP model for each spectrum band. Time is divided into observation intervals of duration T. At the beginning of an observation interval, we start with a BMGPP parameter estimate for each spectrum band and an allocation $\{T_g\}_{g=1}^G$ of the observation interval to subintervals associated with each band such that $\sum_{g=1}^G T_g = T$. During subinterval i, observation samples are collected from spectrum band i, which are used to compute a new BMGPP parameter estimate for the band. The OCBA scheme is applied to allocate an *optimal* budget of subinterval durations $\{T_g\}_{g=1}^G$, which are used to partition the *next* observation interval of duration T into G subintervals. In the multiband spectrum sensing scenario, optimality is with respect to determining the $G^* < G$ bands with the largest mean idle times. At end of the observation interval, G new BMGPP parameter estimates are obtained, as well as the subset of G^* best bands. The procedure is then repeated during the next observation interval of duration T.

We extend the basic OCBA scheme by stopping the procedure when the approximate probability of correct selection (APCS) exceeds a threshold. Thereafter, parameter estimation is performed only for the G^* best bands and the observation subintervals for these bands are assigned equal durations of length T/G^*. We refer to this multiband parameter estimation scheme as `OCBA+EQUAL` and summarize it as follows [9]:

1) *Initialization:* Set the value of $G^* < G$, a threshold probability p^* and the observation interval index $k = 1$. Equally allocate the observation subinterval durations, i.e., $T_1^{(1)} = \cdots = T_G^{(1)} = T/G$. Let $\Sigma_i^{(k)}$ denote the total time allocated to band i up to and including the kth observation interval. Hence, $\Sigma_1^{(1)} = \cdots = \Sigma_G^{(1)} = T/G$.
2) Estimate the mean $\hat{J}_g$ and standard deviation $\hat{s}_g(J)$ of the cost function J.
3) Sort $\{\hat{J}_1, \hat{J}_2, \ldots, \hat{J}_G\}$ such that $\hat{J}_{i_1} \le \hat{J}_{i_2} \le \cdots \le \hat{J}_{i_G}$. Then, the subset $\mathcal{G}^* := \{i_1, i_2, \ldots, i_{G^*}\}$ is the optimal subset with the minimum cost.
4) Compute APCS as follows:

$$\text{APCS} := \prod_{g\in\mathcal{G}^*} \mathsf{P}\{\hat{J}_g \le c\} \cdot \prod_{g\notin\mathcal{G}^*} \mathsf{P}\{\hat{J}_g \ge c\}, \tag{25}$$

where

$$c := \frac{\hat{s}_{i_{G^*+1}}(J)\hat{J}_{i_G} + \hat{s}_{i_{G^*}}(J)\hat{J}_{i_{G^*+1}}}{\hat{s}_{i_{G^*}}(J) + \hat{s}_{i_{G^*+1}}(J)}. \tag{26}$$

5) If $\text{APCS} > p^*$, stop OCBA and continue parameter estimation for set $\mathcal{G}^*$ of selected channels with equal observation subinterval allocation over the G^* bands in $\mathcal{G}^*$; otherwise, continue with Step 6.

6) Compute Σ_i^{k+1}, $i \in \mathcal{G}$, by solving the equations:

$$\frac{\Sigma_1^{(k+1)}}{\left(\hat{s}_1(J)/(\hat{J}_1 - c)\right)^2} = \ldots = \frac{\Sigma_G^{(k+1)}}{\left(\hat{s}_G(J)/(\hat{J}_G - c)\right)^2}, \tag{27}$$

$$\sum_{g=1}^{G} \left(\Sigma_g^{(k+1)} - \Sigma_g^{(k)}\right) = T. \tag{28}$$

7) Compute the next subinterval time allocation:

$$T_g^{(k+1)} = \max\left\{\Sigma_g^{(k+1)} - \Sigma_g^{(k)}, 0\right\},\ g = 1, \ldots, G. \tag{29}$$

8) Increment k by 1. Go to step 2.

B. Estimation of the mean and standard deviation of J

A critical step of OCBA is to estimate the mean and standard deviation of the objective function J. For simplicity, we drop the subscript g and focus on one PU channel. Recall that the sojourn time of state X of a BMMGP or a bivariate continuous-time Markov chain is phase-type distributed, and that ν_a denotes the stationary distribution of the sampled bivariate Markov chain associated to the BMMGP given $X = a$. The probability density function of the sojourn time τ given the state X is obtained as follows (cf. [14]):

$$p_\tau(t \mid X = a) = -\tilde{\nu}_a e^{Q_{aa}t} Q_{aa}\mathbf{1}, \tag{30}$$

where $\tilde{\nu}_a = \frac{\nu_a}{\nu_a \mathbf{1}}$. The mean and variance of the sojourn time τ given $X = a$ are given, respectively, by

$$\mathsf{E}[\tau \mid X = a] = d_a = -\tilde{\nu}_a Q_{aa}^{-1}\mathbf{1}, \tag{31}$$

$$\sigma^2(\tau \mid X = a) = 2\tilde{\nu}_a Q_{aa}^{-2}\mathbf{1} - (\tilde{\nu}_a Q_{aa}^{-1}\mathbf{1})^2. \tag{32}$$

In practice, the true conditional mean and variance are unknown. Instead, we use a finite sequence of observations $\{Y_k\}$ to compute the sample mean and variance of the sojourn time. Given the estimated parameter $\hat{\phi}$ obtained by the EM algorithm in Section II-C, the sample mean and sample variance of the idle time are given, respectively, by

$$\bar{d}_1 = -\hat{\tilde{\nu}}_1 \hat{Q}_{11}^{-1}\mathbf{1}, \tag{33}$$

$$s^2(\tau \mid X = 1) = 2\hat{\tilde{\nu}}_1 \hat{Q}_{11}^{-2}\mathbf{1} - (\hat{\tilde{\nu}}_1 \hat{Q}_{11}^{-1}\mathbf{1})^2. \tag{34}$$

The mean of J is given by

$$\hat{J} = \mathsf{E}[-\bar{d}_1] = -\bar{d}_1 = \hat{\tilde{\nu}}_1 \hat{Q}_{11}^{-1}\mathbf{1}, \tag{35}$$

and the standard deviation of J is given by

$$\hat{s}(J) = s(-\bar{d}_1) = \frac{s(\tau \mid X = 1)}{\sqrt{N_{p1}}}, \tag{36}$$

where N_{p1} denotes the number of idle periods during the observation interval. For a large number of observations, the value of N_{p1} can be approximated by the number of jumps of state X from 2 to 1, which is given by the sum of all elements of the lower left submatrix $\hat{M}^{21}$ in $\hat{M}$, obtained using (23).

IV. Numerical Results

We present some numerical results to demonstrate the performance of the `OCBA+EQUAL` scheme for multiband parameter estimation of BMMGPs. This scheme applies OCBA initially and then, after a stopping criterion is met, allocates observation time equally to each of the bands.

To generate an observation sequence as ground truth in our simulation experiments, the generator matrix Q of a BMMGP was obtained by estimating the parameter of an observation sequence that was collected on the paging band centered at 931.888 MHz by the Shared Spectrum Company [17]. This experimental data was also used in the numerical study in [5]. Parameter estimation was carried out assuming that $r = 10$. The sojourn time distribution of the signal on this band is non-exponentially distributed (see [5, Fig. 5]), which allows us to assess the benefit of using the BMMGP model for parameter estimation. We set the signal-to-noise ratio (SNR) of the received signal to 10 dB and average over 5 energy samples for each observation in all experiments. The sample rate was specified as 26 Hz such that $h = 0.0385$ s. From (31), the true mean sojourn time of idle state can be computed as 0.063 s by the original Q.

A. Mean Idle Time Estimation

We conducted a simulation experiment to evaluate the performance of sojourn time estimation for idle state of a BMMGP with different values of r. For each value of r, we ran 100 simulation trials. The mean idle times from the simulation trials are averaged to obtain the final mean idle time estimate. For each simulation trial, we generated 2000 observations based on the original generator matrix Q with $r = 10$. Figure 1 shows the mean idle time estimate for $r = 1, 2, \ldots, 10$. The red horizontal line shows the true value of mean idle time computed from the original generator matrix, while the blue curve with circle marks shows the estimated mean idle times obtained with different value of r. From the figure, we see that the mean idle time estimates approach the true value as the value of r increases. When $r \geq 5$, the mean idle time estimate is very close to the true value. These results show that modeling the PU channel using a BMMGP can improve the accuracy of parameter estimation significantly compared to that using the simpler MMGP ($r = 1$) when the sojourn time is not exponentially distributed.

B. Probability of Correct Selection

Our next experiment evaluates the probability of correct selection (PCS) for mutiband parameter estimation using different observation time allocation schemes and various values of r. We set up $G = 10$ PU channels with different generator matrices. We selected a random number uniformly distributed in the interval $[a, b]$, and added this number to the largest element in the 10×10 upper right submatrix Q_{12} in the original generator matrix Q. Generally, the value of the modified element should not be negative. In this experiment, we set $a = -75$ and $b = 75$. We then adjusted the associated diagonal element in the upper left submatrix Q_{11} according to (2).

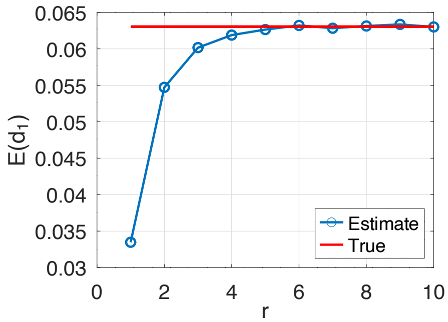

Fig. 1. The estimated mean idle time with different values of r.

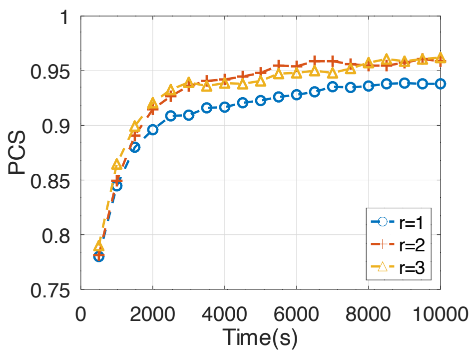

Fig. 2. PCS of `OCBA+EQUAL` scheme with $r = 1, 2, 3$.

Following this procedure, we were able to vary the true mean idle time for each PU channel in the interval $[0.06, 0.074]$. We performed 500 independent simulation runs and averaged the results to obtain smooth curves. Each run contained 20 observation iterations and each observation interval was $T = 500$ s such that the initial allocation to each PU channel was 50 s. For OCBA, we selected the $G^* = 3$ best channels and set the APCS threshold to $p^* = 0.98$.

Figure 2 compares the PCS among the cases of $r = 1, 2, 3$ for the `OCBA+EQUAL` scheme. The PCS in the case of $r = 2$ shows a substantial improvement relative to the case $r = 1$, which shows that using the BMMGP model can achieve better performance in terms of selecting the best subset of channels compared to the MMGP. However, the case $r = 3$ shows little gain relative to the case $r = 2$. Thus, most of the performance gain in OCBA due to more accurate modeling of the sojourn times can be achieved with a small value of r.

V. Conclusion

We proposed a multiband parameter estimation for spectrum sensing based on a bivariate Markov modulated Gaussian process model of PU transmission on a given band. The proposed scheme extends an earlier multiband sensing scheme [9], which combined parameter estimation and computing budget allocation, using an MMGP model of PU transmissions. A drawback of the MMGP-based scheme is that the underlying model implicitly assumes exponentially distributed state sojourn time distributions of the PU transmission process, which is not generally the case in practice.

We developed an EM algorithm for estimating the parameter of a BMMGP. To select the subset of PU channels with the largest mean idle times, we applied OCBA with mean idle time as the objective function. As the mean idle time of a BMMGP is phase-type distributed, non-exponential state sojourn times can be modeled more accurately using the BMMGP compared to the MMGP. Our numerical results confirmed that significantly better accuracy can be achieved via the BMMGP model compared to using the MMGP model. This in turn leads to higher probability of correct selection (PCS) when determining the best subset of G^* bands out of a total of $G \gg G^*$ bands across a wide spectrum. To achieve this gain in PCS, only a small number of additional states in the BMMGP model relative to the MMGP model were needed, which suggests that the extra computational overhead can be kept small in practical scenarios.

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