Peakedness of Stochastic Models for High-Speed Network Traffic

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Abstract — Peakedness was originally developed by teletraffic engineers as a tool for characterizing call arrival processes at a trunk group. We generalize the peakedness theory to include a class of stochastic models used in studies of high-speed networks and apply it to the approximate analysis of a statistical multiplexer.

I. INTRODUCTION

In networks based on the Asynchronous Transfer Mode (ATM), information is transmitted asynchronously over high-speed links in the form of 53-byte units called cells. Accurate traffic characterization is a crucial step in performing network resource allocation and dimensioning.

II. GENERALIZED ARRIVAL PROCESS

Define a rate process \{R_t, t > 0\} to be a strictly stationary random process with finite, nontrivial first two moment measures. The process \{R_t, t > 0\} is to be understood in the generalized function sense with the interpretation that \( R_t \) represents the amount of work arriving in the infinitesimal interval \([t, t + dt]\). The generalized arrival process is then defined by

\[ N_t = \int_0^t R_r \, dr, \]  

where \( N_t \) represents the amount of work arriving in the interval \([0, t]\).

The standard arrival process defined as a stationary point process is a special case with

\[ R_t = \sum_{i=1}^{\infty} b_i \delta(t - T_i), \]  

where \( b_i \) is the number of arrivals at the \( i \)th arrival epoch, \( T_i \), and \( \delta(t) \) is the Dirac delta function. Another special case is the discrete-level fluid process with

\[ R_t = \sum_{i=1}^{\infty} f_i \text{rect}\left(\frac{t - T_i}{T_{i+1} - T_i}\right), \]  

where \( f_i \) is the fluid flow rate, \( T_i \) is the epoch of the \( i \)th transition and \( \text{rect}(t) = u(t) - u(t - 1) \), where \( u(t) \) is the unit step function.

III. GENERALIZED PEAKEDNESS

We introduce a concept of peakedness for a general arrival process as defined by (1). The arrival process is offered to an infinite server system which is represented by an i.i.d. process, \( \{D_t, t > 0\} \), with marginal cdf \( F \). Define

\[ S_t = \int_0^t 1_{\{D_u > t-u\}} R_u \, du, \]  

with the following interpretation: In the interval \([u, u + du]\), \( R_u \, du \) units of work are offered to a new server, introduced at time \( u \), which removes this work from the system after a duration \( D_u \). Then \( S_t \) represents the amount of work present in the system at time \( t \). The peakedness functional with respect to the service time cdf \( F \) is defined by

\[ z[F] = \lim_{t \to \infty} \frac{\text{Var}[S_t]}{E[S_t]^2}. \]  

For the case of an orderly point process, the definitions (4) and (5) reduce to the standard concept of peakedness.

The following result of Eckberg [1] extends to our generalized notion of peakedness:

\[ z[F] = \frac{1 + \mu}{\lambda} \int_{-\infty}^{\infty} \lambda \delta(x - \lambda(x)) F^*(x) dx. \]  

Here, \( F^* \) is the autocorrelation function of \( F^* = 1 - F \), \( \mu^{-1} = \int_{-\infty}^{\infty} F^*(x) dx \) is the mean service time, \( \lambda = E[R_t] \) is the mean arrival rate, and \( k(r) = \text{Cov}(R_{t+r}, R_t) \) is the covariance function of the rate process.

IV. APPLICATION

The generalized peakedness can be obtained in closed form via (6) for a large class of stochastic traffic models, including the popular Markov modulated fluid models. In particular, the peakedness function of a Markov on-off fluid with peak rate \( r \), mean on time \( \beta^{-1} \) and mean off time \( \alpha^{-1} \) with respect to constant service time distribution is given by

\[ z_{\text{const}}(\mu) = \frac{2r^2}{(\alpha + \beta)} \left[ \alpha + \beta + \mu (1 - e^{-\lambda(\alpha + \beta)/\mu}) \right]. \]  

Peakedness can also be estimated empirically through measurements of an actual traffic stream and then used to construct a stochastic traffic model.

Lee and Mark [2] propose a method for approximating a general arrival process with a more computationally tractable superposition of two types of on-off Markov fluid sources by matching central moments of the rate process \( R_t \) and an index of dispersion measure. Since the peakedness function contains strictly more information about the arrival process than the index of dispersion, a more accurate traffic characterization can be achieved by using the peakedness function (7) to perform the match. We demonstrate the effectiveness of our approach with an application to the analysis of a statistical multiplexer.

REFERENCES
