2017 IEEE 56th Annual Conference on Decision and Control (CDC)
December 12-15, 2017, Melbourne, Australia

Multi-agent coordination via a shared wireless spectrum

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Abstract— **This paper considers a planar multi-agent coordination problem. Unlike other related works, we explicitly consider a globally shared wireless communication channel where individual agents must choose both a frequency and power to transmit their messages at. This problem is motivated by the pressing need for algorithms that are able to efficiently and reliably operate on overcrowded wireless networks or otherwise poor-performing RF environments. We develop a self-triggered coordination algorithm that guarantees convergence to the desired set of states with probability 1. The algorithm is developed by using ideas from event/self-triggered coordination and allows agents to autonomously decide for themselves when to broadcast information, at which frequency and power, and how to move based on information received from other agents in the network. Simulations illustrate our results.**

I. Introduction

This paper considers a multi-agent coordination problem where agents can only communicate with one another by broadcasting radio signals over a shared wireless spectrum. This is in contrast to the overwhelming majority of similar works that assume direct point-to-point communications are possible at all times with the possibility of independent chances of packet drops. Instead, we consider a model where agents must not only determine when to communicate with their neighbors, but also what frequency and power their signal should be broadcast on. This model is motivated by the RF spectrum already becoming overcrowded with estimates that there will be 250 times more demand from the spectrum by the year 2030 [1]. In the context of wireless networked cyber-physical systems as a whole, this requires a new generation of coordination algorithms that can (i) *efficiently* utilize the scarce system while being (ii) *resilient* to the fact that communication may not always be possible.

Literature review: In terms of multi-agent coordination, there is a considerable amount of available literature, see e.g., [2], [3], [4]. A continuous-time algorithm that achieves asymptotic convergence to average consensus for both undirected and weight-balanced directed graphs is introduced in [5]. To incorporate the idea of reducing communication, the works [6], [7] use event-triggered broadcasting. More recently, these works have been extended to arbitrary directed graphs, rather than only undirected ones [8], [9], [10]. However, these works all assume a fixed communication topology with direct point-to-point communications on demand. Instead, the same type of coordination problem is considered with a cloud communication model in [11], [12], [13], where agents are only able to communicate indirectly through the use of a central cloud. However, in the above works, this communication is always reliable when needed.

Instead, our communication model is based off of works on Cognitive Radio Networks (CRNs), Dynamic Spectrum Access (DSA), and NeXt Generation (xG) networks [14]. In particular, we are interested in works that consider dynamic allocation of the spectrum [15], [16]. There are a myriad of works on this topic that propose various methods for sensing the spectrum in real time and selecting channels [17], [18], [19], both in centralized and distributed ways. However, these works base their decisions on real-time sensing of the spectrum rather than through coordination or collaboration among the users. Instead, we are interested in the coupling of this type of communication model with an underlying multi-agent/cyber-physical coordination problem, to find conditions to guarantee reliability of the overall system even under poor RF conditions. This idea of utilizing collaboration to help overcome scarcity in the RF spectrum is already of large interest as corroborated by the currently ongoing DARPA Spectrum Collaboration Challenge [20].

Perhaps most similar to the problem we consider is found in [21], where the authors consider the resource management of a single wireless channel and determine exactly when agents should transmit or listen, assuming both cannot be done simultaneously. Instead, we consider a multiple channel communication model where agents are assumed to be listening at all times, but must decide for themselves exactly when, which frequency, and what power to broadcast signals on. Our solution methods are based on ideas from event-triggered and self-triggered control, where algorithms are designed to determine precisely when actions like broadcasting signals or updating actuators should be carried out to improve efficiency while maintaining stability and correctness [22], [23], [24], [25], [26].

Statement of contributions: First, we formulate the coordination problem with a novel communication model that explicitly considers the globally shared wireless spectrum. Then, we develop a distributed self-triggered coordination algorithm that guarantees convergence to a consensus state with probability 1; however, this algorithm is not guaranteed to converge to the exact average of initial states. In particular, this paper is concerned with the minimum conditions required to guarantee convergence to a consensus state. The subject of designing *efficient* algorithms to better utilize the wireless spectrum through collaboration and more efficiently achieve consensus will be discussed in a separate paper.

Preliminaries: The Euclidean norm on $\mathbb{R}^N$ is denoted by $\|\cdot\|$. Given two matrices $A \in \mathbb{R}^{n_1 \times n_1}, B \in \mathbb{R}^{n_2 \times n_2}$, $A \otimes B \in \mathbb{R}^{n_1 n_2 \times n_1 n_2}$ denotes their Kronecker product. The N-dimensional identity matrix is denoted I_N. An undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is defined by a set of vertices $\mathcal{V} = \{1, \dots, N\}$ and edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. Undirected means that $(i,j) \in \mathcal{E}$ if and only if $(j,i) \in \mathcal{E}$ also. An edge $(i,j) \in \mathcal{E}$ means that vertex j is a neighbor of i. The set of neighbors of a given node i is given by $\mathcal{N}_i$. The adjacency matrix $A \in \mathbb{R}^{N \times N}$ is defined by $a_{ij} = 1$ if $(i,j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. A path from vertex i to j is an ordered sequence of vertices such that each intermediate

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978-1-5090-2873-3/17/$31.00 ©2017 IEEE

pair of vertices is an edge. A graph $\mathcal{G}$ is connected if there exists a path from all $i \in \mathcal{V}$ to all $j \in \mathcal{V}$. The degree matrix D is a diagonal matrix where $d_{ii} = |\mathcal{N}_i|$. The maximum degree is $d_{\max} = \max_{i\in\{1,\dots,N\}} |\mathcal{N}_i|$. The Laplacian matrix is a symmetric, positive semidefinite matrix given by $L = D - A$. If the graph $\mathcal{G}$ is connected, the associated Laplacian has exactly one eigenvalue equal to 0 (with associated eigenvector $\mathbf{1}_N$) with the rest being strictly positive, $0 = \lambda_1(L) < \lambda_2(L) \le \dots \le \lambda_N(L)$.

II. Problem Formulation

Consider the N agent coordination problem on the plane, where agent i's position is given by $p_i(t) \in \mathbb{R}^2$ with single-integrator dynamics

$$\dot{p}_i(t) = u_i(t) \tag{1}$$

for all $i \in \{1,\dots,N\}$, where we are interested in reaching a configuration such that $\|p_i(t) - p_j(t)\| \to 0$ as $t \to \infty$ for all $i, j \in \{1,\dots,N\}$. We only consider planar states for simplicity, but note that the results are extendable to arbitrary dimensions given appropriate communication models.

Given a connected interaction graph $\mathcal{G}$, it is well known [5] that the distributed continuous control law

$$u_i(t) = -\sum_{j\in\mathcal{N}_i} (p_i(t) - p_j(t)) \tag{2}$$

drives each agent of the system to asymptotically converge to the average of the agents' initial conditions. In compact form, this can be expressed by

$$\dot{p} = -\overline{L}p,$$

where $p = (p_1^T,\dots,p_N^T)^T$ is the column vector of all agent states and $\overline{L} = L \otimes I_2$. However, this control law requires each agent to have exact information about its neighbors at all times and continuously update its control law.

There have been many recent works aimed at relaxing one or both of these requirements [8], [27], [10], [6]. However, they all require events triggered by some agent to be immediately acknowledged by neighboring agents. In other words, when an event is triggered by one agent, its neighbors are immediately aware and can take action accordingly. The authors in [7] consider this problem in the presence of unreliable communication but assume that packet drops randomly occur independent of what is actually going on in the network. Instead, we are interested in explicitly modeling the fact that packet drops occur because of interference with other agents in the system.

In all of these works (except [7]), they utilize a piecewise constant implementation of the controller (2) given by

$$u_i(t) = -\sum_{j\in\mathcal{N}_i} (\widehat{p}_i(t) - \widehat{p}_j(t)), \tag{3}$$

where $\widehat{p}_j(t)$ is the last broadcast position of agent j. Note that although agent i has access to its own state $p_i(t)$, the controller (3) uses the last broadcast state $\widehat{p}_i(t)$. This is to ensure that the average of the agents' initial states is preserved throughout the evolution of the system. This means that when an agent i broadcasts its current position $p_i(t^*)$ at some time t^*, all its neighbor $j \in \mathcal{N}_i$ immediately update $\widehat{p}_i = p_i(t^*)$ so that the average can be preserved.

Instead, in this work, we are interested in explicitly modeling wireless interference depending on how different agents in the network are broadcasting messages. To simplify the communication model later, we consider a discrete-time implementation of the dynamics (1) given by

$$p_i(\ell+1) = p_i(\ell) + hu_i(\ell), \tag{4}$$

where $h > 0$ is the timestep.

Unlike all the similar works mentioned above, we consider a new communication model that is motivated by the overcrowding of the RF spectrum. We describe the communication model next.

Communication via a Shared Wireless Spectrum

In order to communicate wirelessly, the agents will have to broadcast signals with a chosen power and frequency among a shared spectrum. We denote by $S = \{s_1,\dots,s_M\}$ the set of M different frequency bands, or channels, available for wireless communication. Since signals at the same frequency can still be successfully transmitted and received when considering spatial reuse, the whole shared spectrum state at any given timestep ℓ is given by the Cartesian product of physical space and frequency $\mathcal{S} = \mathbb{R}^2 \times S$.

In order for these signals to be successfully received, they will have to be received with a sufficient level of power without interference. We now define exactly what subset of the spectrum $\mathcal{S}$ is considered 'occupied' at a given time ℓ given full broadcast information about the network.

At any given time $\ell \in \mathbb{Z}_{\ge 0}$, we denote by $\mathcal{B}_i(\ell) = (s_i(\ell), P_i(\ell)) \in S \times \mathbb{R}_{\ge 0}$ the current broadcast state of agent i, where $s_i(\ell) \in S$ and $P_i(\ell) \in \mathbb{R}_{\ge 0}$ are the chosen frequency and power of the signal, respectively. For simplicity, we may drop the explicit dependence on time ℓ.

Given an agent i's current position p_i and broadcast state $\mathcal{B}_i$, we assume that the signal can be successfully received by any agent j if it is within $r(P_i)$ of agent i, where $r : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is a nondecreasing function of the broadcast power, *and* there is no signal interference at node j's location. Given p_i and $\mathcal{B}_i$, we can then compute the portion of the shared spectrum $\mathcal{S}$ that is being used by

$$O_i(p_i, \mathcal{B}_i) = \{(p', s_i) \in \mathcal{S} \mid \|p' - p_i(\ell)\| \le r(P_i), \},$$

With this, we can compute the entire occupied subset of the spectrum $\mathcal{O} \subset \mathcal{S}$ at any given time by

$$\mathcal{O} = \bigcup_{i\in\{1,\dots,N\}} O_i(p_i, \mathcal{B}_i).$$

Similarly, we denote by

$$\mathcal{O}^{-i} = \bigcup_{j\ne i} O_j(p_j, \mathcal{B}_j)$$

the portion of the spectrum being occupied by any agent besides i. More specifically, $\mathcal{O}^{-i} \subset \mathcal{S}$ is the set of physical locations and frequencies at which a signal broadcast by agent i *cannot* be received due to RF interference.

With this, given an agent i's current position p_i, broadcast state $\mathcal{B}_i$, and the current spectrum occupancy $\mathcal{O}$, the set of locations in $\mathbb{R}^2$ that can successfully receive this transmission is given by

$$\mathcal{R}(p_i, \mathcal{B}_i, \mathcal{O}) = \underbrace{\overline{B}(p_i, r(P_i))}_{\text{points that signal can reach}} \setminus \underbrace{\text{loc}(\mathcal{O}^{-i}, s_i)}_{\text{points with RF interference}},$$

$p_i = (x_i, y_i)$	$\in \mathbb{R}^2$	position
u_i	$\in \mathbb{R}^2$	control input
$\mathcal{B}_i = (s_i, P_i)$	$\in S \times \mathbb{R}_{\geq 0}$	broadcast state
s_i	$\in S$	frequency of signal
P_i	$\in \mathbb{R}_{\geq 0}$	power of signal
$O_i(p_i, \mathcal{B}_i)$	$\in \mathcal{S}$	occupied portion of spectrum
$\widetilde{p}^i_j$	$\in \mathbb{R}^2$	last received position of agent j

TABLE I
AGENT i MODEL DEFINITIONS.

where $\overline{B}(p_i, r(P_i)) = \{p \in \mathbb{R}^2 \mid \|p_i - p\| \leq r(P_i)\}$ denotes the closed ball centered at p_i with radius $r(P_i)$, and

$$\text{loc}(\mathcal{O}^{-i}, s_i) = \{p' \in \mathbb{R}^2 \mid (p', s_i) \in \mathcal{O}^{-i}\}$$

extracts the physical locations from $\mathcal{O}^{-i}$ where interference exists at frequency s_i. With a slight abuse of notation, we let $\mathcal{R}(p_i, \mathcal{B}_i, \mathcal{O}) = \emptyset$ whenever $P_i = 0$. In other words, if a signal is broadcast with zero power, the set of points that can successfully receive this signal is empty. Thus, in order for agent j to successfully receive a message from agent i at time ℓ, we need $p_j(\ell) \in \mathcal{R}(p_i(\ell), \mathcal{B}_i(\ell), \mathcal{O}(\ell))$, which means agent j is close enough to agent i to receive the signal *and* there is no RF interference from any other agent at that time. Note that we assume all agents can receive signals at any frequency $s \in S$.

Equipped with this model, we define $\widetilde{p}^i_j(\ell)$ as the last successfully received position of agent j by agent i up to time ℓ. More specifically, for neighbors $j \in \mathcal{N}_i$,

$$\widetilde{p}^i_j(\ell) = \begin{cases} p_j(\ell) & \text{if } p_i(\ell) \in \mathcal{R}(p_j(\ell), \mathcal{B}_j(\ell), \mathcal{O}(\ell)), \\ \widetilde{p}^i_j(\ell - 1) & \text{otherwise.} \end{cases} \tag{5}$$

Thus, $\widetilde{p}^i_j(\ell)$ is updated with agent j's true position $p_j(\ell)$ only if agent i is in a location that was able to successfully receive the signal $\mathcal{B}_j(\ell)$ broadcast by agent j.

Based on the information $\widetilde{p}^i_j(\ell)$ available to agent i at time ℓ, it must then determine a control input for the dynamics (4). These objects are summarized in Table I. The full problem is formalized below.

Problem II.1 Let $\widetilde{p}^i_j(0) = p_j(0)$ for all $i \in \{1, \dots, N\}, j \in \mathcal{N}_i$. Given the discrete-time dynamics (4) with the communication model (5), find a distributed communication and control strategy $\{u_i(\ell), \mathcal{B}_i(\ell)\}$ for all agents such that $\lim_{\ell \to \infty} \|p_i(\ell) - p_j(\ell)\| = 0$ for all $i, j \in \{1, \dots, N\}$.

To summarize, in each timestep $\ell \in \mathbb{Z}_{\geq 0}$, each agent i must determine whether to broadcast a signal or not, what frequency and power to broadcast at, and what local control input to use, such that the closed-loop system converges.

Remark II.2 (Interaction vs. communication) It should be noted that our communication model allows agents to communicate independent of the fixed interaction graph $\mathcal{G}$, so an agent i might have information about any subset of the agents rather than only the neighbors in $\mathcal{G}$. •

Remark II.3 (Multi-agent formation control) We note here that for simplicity, we formulate the multi-agent coordination problem as a consensus problem. The formal treatment can easily be modified to handle a formation control problem by letting $p_i(t) = \Delta p_i(t) - b_i$, where $\Delta p_i(t)$ is the actual position of agent i and b_i is the desired displacement from the average position of the fleet. Alternatively, the interaction graph $\mathcal{G}$ could also be used to encode a desired formation into the consensus problem. •

III. SELF-TRIGGERED ALGORITHM DESIGN

Having fully defined the problem, we first point out that if we remove the problem of determining a frequency and power to broadcast a signal on, the problem becomes very similar to those studied in many different works considering event-triggered consensus. In particular, these works study when agents should broadcast state information to each other and how to update their control signals. More specifically, the motion control algorithm used in many event-triggered consensus problems [6], [7], [9], [8], [10] is given by

$$u_i(\ell) = -\sum_{j \in \mathcal{N}_i} (\widehat{p}_i(\ell) - \widehat{p}_j(\ell)), \tag{6}$$

where $\widehat{p}_i(\ell)$ is the last state broadcast by agent i at time ℓ. Note that this motion control algorithm implicitly assumes that any message broadcast by an agent i will be successfully received by its neighbors $j \in \mathcal{N}_i$, e.g., $\widetilde{p}^i_j(\ell) = \widehat{p}_j(\ell)$ for all $i \in \{1, \dots, N\}$ and $\ell \in \mathbb{Z}_{\geq 0}$. Under this assumption, we first extend the main results in [27], [7] to discrete time and the higher order space $\mathbb{R}^2$.

Lemma III.1 *Given the dynamics* (4) *with control law* (6)*, if the sequence of broadcast times* $\{\ell^i_k\}_{k \in \mathbb{Z}_{\geq 0}}$ *for all agents* $i \in \{1, \dots, N\}$ *guarantees that*

$$\|p_i - \widehat{p}_i\|^2 \leq D_i \sum_{j \in \mathcal{N}_i} \|\widehat{p}_i - \widehat{p}_j\|^2,$$

for all ℓ *with* $D_i = \sigma_i \frac{a}{|\mathcal{N}_i|}\left(1 - \frac{h}{2} - a|\mathcal{N}_i|\right)$ *for some* $a \in \left(0, \frac{h}{2d_{max}}\right)$ *and* $\sigma_i \in (0, 1)$*, then the system will asymptotically converge to the average consensus state. In other words,* $p_i(\ell) \to \text{Avg}(p(0))$ *as* $\ell \to \infty$ *for all* $i \in \{1, \dots, N\}$.

Lemma III.1 provides a sufficient event-trigger for convergence that determines when agents should broadcast their states assuming no packet drops or other wireless interference. This result is useful as these prior works study exactly when broadcasting state information is necessary in order to reduce the amount of communication required by the network to achieve consensus.

Unfortunately, the communication and control strategy given in Lemma III.1 assumes that communication is always available when necessary. Instead, under our new communication model and assuming no prior information is shared among the agents, avoiding packet drops completely will be impossible. More specifically, it is very possible that a signal broadcast by an agent j is not received by some neighbor i for two different reasons. The first is that the power used by agent j to send the signal was simply not strong enough to reach agent j, and the second is that a third agent's transmission is interfering with agent j's signal at agent i's location. Consequently, the control law (6) is no longer implementable because this assumes no packet drops or wireless interference occurs.

Motion Control Law

Instead, we consider the control input for agent i at any given time to be

$$u_i(\ell) = -\sum_{j \in \mathcal{N}_i} p_i(\ell) - \widetilde{p}_j^i(\ell), \qquad (7)$$

where $\widetilde{p}_j^i(\ell)$ is the last successfully received position of agent j by agent i given in (5). In other words, $\widetilde{p}_j^i(\ell)$ only gets redefined as $p_j(\ell)$ if agent i was in the set of points $\mathcal{R}(p_j(\ell), \mathcal{B}_j(\ell), \mathcal{O}(\ell)) \subset \mathbb{R}^2$ that could successfully receive the broadcast $\mathcal{B}_j(\ell)$ from location $p_j(\ell)$ with no interference. Another difference here with respect to (6) is that agent i is now using its exact position $p_i(\ell)$ rather than its last broadcast position $\widehat{p}_i(\ell)$. The reason for this is due to the nature of how communication occurs. In particular, agents are only able to decide for themselves when to broadcast signals to other agents, but there is no mechanism for them to request information from them. This means that when an agent i doesn't know how to move because its information is too outdated, it must wait for additional information to arrive before it can begin moving again. Thus, while the agent is waiting, the control law (7) ensures that this agent will eventually stop moving until new information is received. This type of control law is desirable in our scenario because it is guaranteed to not drive the system to instability even if the communication network is completely shutdown.

Interestingly, trajectories of the dynamics (4) with control (7) behave very similarly with consensus algorithms with randomized gossiping [28], [29] or switching topologies [5]. Consequently, we are able to provide a result to guarantee convergence as long as enough successful transmissions occur. With a slight abuse of notation, we define $\{\ell_{k'}^{i \to j}\}_{k' \in \mathbb{Z}_{\geq 0}} \subset \{\ell_k^i\}_{k \in \mathbb{Z}_{\geq 0}}$ as the sequence of times that a message broadcast by agent i was successfully received by agent j. Then, given the communication model (5),

$$\{\ell_{k'}^{i \to j}\}_{k' \in \mathbb{Z}_{\geq 0}} = \{\ell_{k'}^i \mid p_j(\ell_{k'}^i) \in \mathcal{R}(p_i(\ell_{k'}^i), \mathcal{B}_i(\ell_{k'}^i), \mathcal{O}(\ell_{k'}^i))\},$$

is the subset of times at which agent i broadcast a message that was successfully received by another agent j. A condition for convergence is then formalized in Theorem III.2.

Theorem III.2 *Given the dynamics* (4) *with control law* (7)*, if* $\ell_{k'}^{i \to j} \to \infty$ *as* $k' \to \infty$ *for all* $i \in \{1, \dots, N\}, j \in \mathcal{N}_i$*, then* $\lim_{\ell \to \infty} \|p_i(\ell) - p_j(\ell)\| = 0$ *for all* $i, j \in \{1, \dots, N\}$.

Communication Strategy

Given the result of Theorem III.2 and the fact that we are only concerned with stability, it is easy to imagine there are many different ways to design communication strategies that satisfy the conditions of Theorem III.2. Due to space restrictions, we only propose one possible strategy and discuss a few of its key features aimed at minimizing how much communication is required while still guaranteeing convergence. Choosing a communication strategy to optimize different metrics are instead studied in a separate paper.

The three components of the communication algorithm we need to design are how agents should determine *when* to communicate, with what *power*, and what *frequency*. To answer the question of when agents should decide to broadcast, inspired by Lemma III.1, we define the self-triggered broadcasting time as

$$\ell_{k+1}^i - \ell_k^i = \left\lceil \frac{\tau_1}{\tau_2 + h \sum_{j \in \mathcal{N}_i} \|\widehat{p}_i - \widetilde{p}_j^i\|^2} \right\rceil,$$

where $\tau_1, \tau_2 > 0$ are design parameters. Equipped with a method for determining when agents should communicate, they also need to choose a power and frequency. In general, it would be ideal for an agent i to use the minimum power required to reach its neighbors,

$$P_i^* = r^{-1}(\max_{j \in \mathcal{N}_i} \|p_i - p_j\|),$$

where $r^{-1}(D)$ is the power required to transmit a message a distance D away. Unfortunately, agent i will not know the positions of its neighbors in general. However, since the goal of the algorithm is to achieve consensus, the power required over time should be decreasing. Unfortunately, we cannot do this blindly since we do not assume any acknowledgement messages, it would be impossible for agent i to know if any message has successfully been received by any agent. To address this, we define an integer $d > 0$, where if an agent i has not successfully received any message from a neighbor $j \in \mathcal{N}_i$ in the time that agent i has broadcast its position d times, it will increase its communication power. This guarantees that, while a neighbor j cannot get a message to i, agent i will keep increasing its communication power until eventually the broadcast can reach agent j. In turn, agent j will receive agent i's position and set its power accordingly. Although this guarantees that a message can eventually be sent with enough power, we still need to choose a frequency. To guarantee that all messages will eventually succeed (even if with a small probability), we utilize a simple jump-stay channel hopping method where an agent i will stick to its last used frequency with probability α and randomly change its frequency with probability $1 - \alpha$. The self-triggered coordination algorithm is formally described in Table II.

For each agent $i \in \{1, \dots, N\}$ at times $\ell \in \mathbb{Z}_{\geq 0}$, performs:

1: receive $\widetilde{p}_j^i = p_j$ from all neighbors j where $p_i \in \mathcal{R}(p_j, \mathcal{B}_j, \mathcal{O})$ and set $\ell^{j \to i} = \ell$
2: **if** $\ell = \ell_k^i$ (this is a scheduled broadcast time) **then**
3:   **if** $\exists j$ such that $\ell^{j \to i} \leq \ell_{k-d}^i$ **then**
4:     set $P_i(\ell) = 1.1 P_i(\ell - 1)$
5:   **else**
6:     set $P_i(\ell) = \min\{P_i(\ell), r^{-1}(\max_j \|p_i - \widetilde{p}_j^i)\|$
7:   **end if**
8:   set $s_i(\ell) = s_i(\ell_{k-1}^i)$ w.p. α, or randomly from S w.p. $1 - \alpha$
9:   broadcast position $p_i(\ell)$ and next scheduled broadcast time ℓ_{k+1}^i at frequency $s_i(\ell)$ and power $P_i(\ell)$
10:   set $k = k + 1$
11: **end if**
12: update control $u_i(\ell)$ according to (7)

TABLE II
SELF-TRIGGERED COORDINATION ALGORITHM.

The main convergence result, formally stated below, is then a direct consequence of Theorem III.2 and the fact that the self-triggered coordination algorithm ensures all agents will attempt to communicate with their neighbors with sufficient power an infinite number of times.

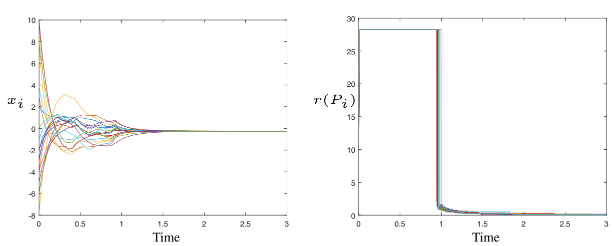
Theorem III.3 *The self-triggered coordination algorithm guarantees asymptotic convergence to the set of consensus states with probability 1, i.e.,* $\|p_i(\ell) - p_j(\ell)\| \to 0$ *for all* $i, j \in \{1, \ldots, N\}$ *as* $\ell \to \infty$ *with probability 1.*

It is clear and known that randomly selecting a channel is not efficient [30] and efficiently selecting channels based on real-time spectrum sensing is currently an active area of research [31], [17], [18]. Instead, we are interested in using collaboration between agents to better utilize the spectrum; however, as this paper is only concerned with stability this will be discussed in detail in a separate paper.

IV. Simulations

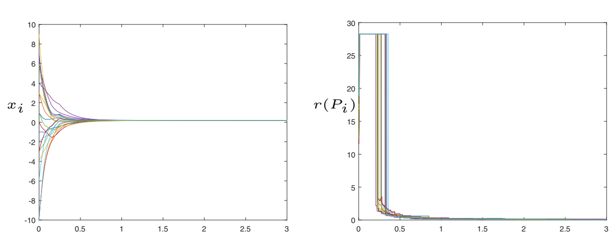
Due to the limited space and the fact that we have not yet discussed communication strategies designed to optimize various efficiency metrics, we only show two executions of the proposed algorithm for a system of $N = 20$ agents with $d = 5, \alpha = 0.5, h = 0.001, \tau_1 = 0.5$, and $\tau_2 = 0.05$ to verify the main convergence result of the paper.

We compare executions of the algorithm with $M = 3$ and $M = 5$ different channels. Figures 1-2(a) show the horizontal component of agent positions over time and Figures 1-2(b) show the power being utilized (capped based on initial condition) by each agent to broadcast signals. It is interesting to note that the plots (b) reveal that the agents are struggling to establish communication to begin with before they are able to well coordinate themselves. Giving them more channels allows them to establish successful communications early on to begin coordinating themselves better. As this paper is only concerned with stability, discussions about performance will be presented in a separate paper.



(a) (b)

Fig. 1. Plots of (a) horizontal component of positions and (b) power allocations over time with $M = 3$ channels and $N = 20$ agents.



(a) (b)

Fig. 2. Plots of (a) horizontal component of positions and (b) power allocations over time with $M = 5$ channels and $N = 20$ agents.

V. Conclusions

This paper formulates and solves a coordination problem where agents communicate through a shared wireless spectrum. We propose a self-triggered coordination algorithm that combines a communication strategy with a motion control law that guarantees convergence with probability 1. The focus of this paper is on minimum conditions for stability under the proposed communication model motivated by the need for coordination algorithms that need to communicate via crowded or otherwise unreliable wireless networks.

Future work will be devoted to actually quantifying the efficiency of the network and formulating appropriate optimization problems to make better use of the scarce spectrum through tight collaboration among the agents. In addition, we also plan to further develop the communication model to capture more realistic scenarios. For instance, we have only considered a synchronous discrete-time TDMA schedule which we plan to relax in the future. We also currently only consider a hard collision model where both packets are dropped when collisions occur. However, by more explicitly considering real-time Signal-Interference-to-Noise Ratio (SINR) when determining collisions, it is possible for interference to only cause one of the two interfering packets to be dropped. This will give agents the ability to essentially ‘shout’ their high-priority messages to increase the chance of it being successfully received at the cost of drowning out communications to nearby agents operating at that frequency. We expect these scenarios to become increasingly important problems that need to be addressed as more and more wireless devices come into existence.

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APPENDIX

Proof of Lemma III.1: For convenience, we define $(x_i, y_i) = p_i$ and $(\widehat{x}_i, \widehat{y}_i) = \widehat{p}_i$. Let $V(\ell) = \frac{1}{2}(x^T Lx + y^T Ly)$. Given the dynamics (4) with the motion control law (6), defining $\Delta V(\ell) = V(\ell+1) - V(\ell)$, we have

$$\Delta V(\ell) = -h(x^T LL\widehat{x} + y^T LL\widehat{y}) + \frac{1}{2}h^2(\|L\widehat{x}\|^2 + \|L\widehat{y}\|^2).$$

Letting $\Delta V_x(\ell) \triangleq \frac{V(\ell+1)-V(\ell)}{h}$ and $e_x = \widehat{x} - x$, we can expand

$$\Delta V_x(\ell) = -\left(1 - \frac{1}{2}h\right)\|L\widehat{x}\|^2 + e_x^T LL\widehat{x}.$$

Defining $\Delta V_y(\ell)$ similarly, note that the sign of $\Delta V(\ell)$ is equivalent to the sign of $\Delta V_x(\ell) + \Delta V_y(\ell)$. Given the similarity of ΔV_x and ΔV_y, we now focus on ΔV_x to simplify the presentation.

Letting $\widehat{z} \triangleq L\widehat{x}$, we can expand out

$$\begin{aligned}\Delta V_x(\ell) &= -\sum_{i=1}^N \left(1 - \frac{1}{2}h\right)\widehat{z}_i^2 + \widehat{z}_i \sum_{j\in\mathcal{N}_i}(e_{x,i} - e_{x,j}) \\ &= -\sum_{i=1}^N \left(1 - \frac{1}{2}h\right)\widehat{z}_i^2 + |\mathcal{N}_i|\widehat{z}_i e_{x,i} - \widehat{z}_i \sum_{j\in\mathcal{N}_i} e_{x,j}.\end{aligned}$$

Using Young's inequality, we can bound

$$\begin{aligned}|\mathcal{N}_i|\widehat{z}_i e_{x,i} &\le \frac{1}{2}|\mathcal{N}_i|\widehat{z}_i^2 a + \frac{1}{2a}|\mathcal{N}_i| e_{x,i}^2, \\ -\widehat{z}_i \sum_{j\in\mathcal{N}_i} e_{x,j} &\le \sum_{j\in\mathcal{N}_i} \frac{1}{2}\widehat{z}_i^2 a + \frac{1}{2a}e_{x,j}^2,\end{aligned}$$

for any $a > 0$. Since the graph is undirected, we have

$$\sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{1}{2a}e_{x,j}^2 = \sum_{i=1}^N \sum_{j\in\mathcal{N}_i} \frac{1}{2a}e_{x,i}^2 = \sum_{i=1}^N \frac{1}{2a}|\mathcal{N}_i| e_{x,i}^2.$$

Combining this with the inequalities above yields

$$\Delta V_x(\ell) \le \sum_{i=1}^N \left(a|\mathcal{N}_i| + \frac{1}{2}h - 1\right)\widehat{z}_i^2 + \frac{|\mathcal{N}_i|}{a}e_{x,i}^2.$$

Choosing a small enough such that $a < \frac{h}{2|\mathcal{N}_i|}$ for all $i \in \{1, \ldots, N\}$, if

$$e_{x,i}^2 \le \sigma_i\left(\frac{a}{|\mathcal{N}_i|}\right)\left(1 - \frac{h}{2} - a|\mathcal{N}_i|\right)\widehat{z}_i^2,$$

is satisfied at all times ℓ, then

$$\Delta V(\ell) \le \sum_{i=1}^N (\sigma_i - 1)\left(1 - \frac{h}{2} - a|\mathcal{N}_i|\right)\widehat{z}_i^2.$$

Combining this with the condition of the Lemma reveals that $\Delta V(\ell) < 0$ as long as there exists i, j such that $\|p_i - p_j\| \neq 0$. The proof is finally concluded by noticing that $\mathrm{Avg}(p(\ell))$ is in invariant quantity under (6). •

Proof of Theorem III.2: Due to space restrictions we only present a simple sketch here. Rather than analyzing p_i directly, we will instead analyze its components (x_i, y_i). Let $x^+(\ell) = \max_i x_i(\ell)$ and $x^-(\ell) = \min_i x_i(\ell)$. Take any i^* such that $x_{i^*} = x^+$, then according to (7)

$$\dot{x}_i^* = -\sum_{j\in\mathcal{N}_{i^*}} x_i - \widetilde{x}_j^i. \tag{8}$$

Noticing that $\widetilde{x}_j^i$ is simply a random, nonuniform sampling of x_j, we can always bound $\widetilde{x}_j^i \in [x^-, x^+]$, which means we can upper-bound (8) by 0. Thus, $x^+(\ell) \le x^+(0)$ for all $\ell \ge 0$. Similarly, $x^- i(\ell) \ge x^-(0)$ for all $\ell \ge 0$. To show that eventually $x^+(\ell) = x^i(\ell)$, notice that if $x^+(\ell) \neq x^i(\ell)$, there exists at least one i^* with $x_i^* = x_i^+$ with a neighbor j such that $x_j < x_i^*$, because the graph is connected. Using this, we see that (8) becomes strictly negative assuming updated information is available. Finally, it is important to note that under (7), in the case that $\widetilde{p}_j^i$ are assumed constant, the convergence of x_i to a fixed point is asymptotic. This rules out the possibility of two agents being able to fully swap places in between successful communications, which concludes the result. •