Collaborative Spectrum Sensing Based on Hidden Bivariate Markov Models

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***Abstract*—The purpose of spectrum sensing is to determine idle portions of a licensed spectrum band that could be used by unlicensed or secondary users without causing harmful interference to primary users. Collaborative spectrum sensing involves multiple secondary users to make joint decisions about spectrum occupancy. By exploiting multiuser diversity, collaborative sensing can alleviate the effects of hidden terminals and severely shadowed radio environments. In this paper, we investigate and compare two schemes for collaborative spectrum sensing of a narrowband channel based on online parameter estimation of a hidden bivariate Markov model: a hard decision scheme and a soft decision scheme. Relative to prior collaborative sensing approaches that do not incorporate a model of the state of the primary user, the proposed schemes improve the accuracy and reliability of collaborative spectrum sensing, especially in low signal-to-noise ratio environments. Numerical results are presented to demonstrate the performance of the proposed model-based collaborative spectrum sensing schemes.**[1]

***Index Terms*—Dynamic spectrum access; cognitive radio; spectrum sensing; collaboration; hidden Markov model; online recursive estimation.**

I. Introduction

Cognitive radio is an emerging technology for reclaiming underutilized spectrum resources, which will likely play a role in future 5G wireless networks. In a wireless network supporting dynamic or opportunistic spectrum access, unlicensed or secondary users (SUs) are permitted to make use of portions of a licensed spectrum band that are left idle by the licensed or primary users (PUs) as long as no harmful interference is incurred on the PUs. In such a scenario, the SUs are equipped with cognitive radios that can detect spectrum holes and make use of such holes for their own communications. Spectrum holes can be characterized in space, frequency, and time. In this paper, we focus on temporal spectrum sensing of a given channel, where the PU occupying the channel alternates between an active and an idle state. The SU attempts to determine the time intervals for which the channel is idle.

Various approaches to temporal spectrum sensing have been studied in the literature, including energy detection, matched filter detection, and cyclostationary detection (cf. [1]). Matched filter detection requires knowledge of the modulation scheme employed by the PU. Cyclostationary detection can provide much better performance than energy detection, but is much more computationally intensive and requires longer sensing times. In [2], an approach to temporal spectrum sensing based on a hidden bivariate Markov model (HBMM) was proposed. The HBMM extends the more traditional hidden Markov model (HMM) by allowing for more general state sojourn time distributions of the PU. The state sojourn times of an HBMM have discrete phase-type distributions, whereas those of an HMM are limited to geometric distributions. The higher degrees of freedom afforded by the HBMM can provide higher modeling fidelity than the HMM, which results in better detection performance. Recently, a recursive parameter estimation algorithm for the HBMM, based on a recursive parameter estimation algorithm developed by Rydén [3], was developed in [4]. Together with the state estimation and prediction recursions presented in [2], the recursive HBMM parameter estimation algorithm forms the basis for fully online temporal spectrum sensing scheme.

In radio environments with severe shadowing and fading effects, spectrum sensing by a single SU can lead to hidden terminal effects and other errors which can result in harmful interference to the PUs. Collaborative or cooperative spectrum sensing techniques leverage multiuser diversity to improve sensing performance. These techniques extend the temporal sensing methods mentioned above by involving multiple SUs in a joint decision-making process to determine when a given channel is idle or active. Collaborative sensing can be categorized into two main approaches: hard decision fusion and soft decision fusion. In hard decision fusion, each SU in a group of SUs makes an independent decision on the active or idle state of the channel. These 1-bit decisions are forwarded to a fusion center, which combines the individual SU decisions into a final decision according to a fusion rule. Two popular fusion rules are the majority vote and the more conservative OR-rule, in which the PU is deemed active if at least one of the SU decides that the PU is active. Hard decision fusion based on voting is one of the simplest suboptimal collaboration methods [5].

In soft decision fusion, each of the SUs forwards measurements to the fusion center, which is combined according to

[1]This work was supported in part by the U.S. National Science Foundation under Grants No. 0916568, No. 1205453, and No. 1421869.

a set of weights and compared to a threshold to determine the final decision. The soft decision fusion scheme uses the likelihood ratio test (LRT) to implement the Neyman-Pearson classifier [6]. This classifier, however, involves a quadratic form leading to high computation cost, and the performance evaluation and threshold computations are also mathematically less tractable. In [7], a linear fusion rule is proposed for a soft decision scheme which optimizes a modified deflection coefficient (MDC) that characterizes the probability distribution function of the global test statistic at the fusion center. This approach has less computational complexity, is more tractable, and achieves performance comparable to the LRT-based fusion. Soft decision schemes can outperform hard decision schemes in terms of detection accuracy, but the weights for soft fusion must be chosen appropriately, and the computational and communication overhead is much higher. Thus, both hard decision and soft decision schemes may be appropriate for spectrum sensing in various scenarios.

In this paper, we develop online hard and soft decision schemes for collaborative spectrum sensing based on the HBMM and associated online parameter estimation algorithms. In conventional collaborative sensing schemes, parameters such as the decision thresholds and weights for soft linear combining are determined offline, based on training data. Our online approach automatically adjusts the thresholds and weights to the appropriate values based on the real-time observation data and thus can adapt to changes in the wireless environment. Compared to the prior collaborative sensing methods that do not employ a model for the temporal dynamics of the PU, the proposed schemes provide superior spectrum detection performance, especially in low signal-to-noise ratio environments. Our approach relies on the online parameter estimation algorithm presented in [4]. As in [2], [4] the detector front-end for each SU is assumed to provide averaged energy estimates obtained from received signal strength measurements, and the channel follows a path loss model with lognormal shadowing.

The remainder of the paper is organized as follows. In Section II, we discuss the system model for collaborative sensing and briefly review the HBMM-based approach for temporal spectrum sensing by a single SU developed in [2], [4]. In Section III, we develop HBMM-based hard decision and soft decision collaborative sensing schemes. In Section IV, we present numerical results to demonstrate the performance of the proposed collaboration schemes. In Section V, we provide concluding remarks.

II. System Model

A. Collaborative sensing model

We consider a system consisting of one PU transmitting on a given narrowband channel and Q SUs performing collaborative spectrum sensing on the channel. The PU alternates between active state in which a signal is transmitted and an idle state in which no signal is transmitted. Meanwhile, each SU performs spectrum sensing and computes the received energy from the PU. The state of the PU is represented by a discrete-time process $X = \{X_k\}$ such that at time k the state of the PU is given by the random variable

$$X_k = \begin{cases} 1, & \text{idle state}, \\ 2, & \text{active state}. \end{cases} \tag{1}$$

Let $Y^{(q)} = \{Y_k^{(q)}\}$ denote the sequence of observable energy measurements obtained at the output of the front-end for the qth SU, $q = 1, \ldots, Q$. Assuming a standard path loss plus lognormal shadowing model, the received signal strength $Y_k^{(q)}$, in units of dBm, can be expressed as follows (cf. [4]):

$$Y_k^{(q)} = \begin{cases} \mu_1^{(q)} + \epsilon_{1,\mathrm{dB}}^{(q)}, & X_k = 1, \\ \mu_2^{(q)} + \epsilon_{2,\mathrm{dB}}^{(q)}, & X_k = 2, \end{cases} \tag{2}$$

where $\mu_a^{(q)}$ represents the mean received signal strength of the qth SU when the PU is in state $a \in \{1, 2\}$ and $\epsilon_{a,\mathrm{dB}}^{(q)}$ is a zero-mean Gaussian random variable with standard deviation $\sigma_a^{(q)}$, which represents lognormal normal shadowing. Let $\boldsymbol{Y} = (Y^{(1)}, \ldots, Y^{(Q)})$ denote the vector process of observables from the Q SUs.

B. Hidden bivariate Markov model

An HBMM is a trivariate process (Y, X, S), where Y denotes an observable process with continuous alphabet and the underlying process, $Z = (X, S)$, is a finite-state bivariate Markov chain. In [4], HBMM was adopted to model the temporal spectrum sensing for a single SU. Here, Y is used to represent the received signal power at an SU and X represents the state of the PU. The process S is introduced so that the sojourn time of the process X in each state $a \in \{1, 2\}$ takes on a discrete-time phase-type distribution.

For a general HBMM, we denote the state-space of X by $\mathbb{X} = \{1, \ldots, d\}$, the state-space of S by $\mathbb{S} = \{1, \ldots, r\}$, and we let $\mathbb{Z} = \mathbb{X} \times \mathbb{S}$ denote the state-space of Z. The processes Y and S are assumed to be conditionally independent given X. Let $f(y_k; \theta_a)$ denote the conditional density of Y_k given $X_k = a$ at time k, where θ_a is a parameter depending on $a \in \mathbb{X}$. From (2), $f(y_k; \theta_a)$ is a Gaussian density and we set $\theta_a = (\mu_a, \sigma_a)$ to be the mean and standard deviation of this density. The initial distribution of Z is denoted by a $1 \times dr$ row vector $\pi = [\pi_{(a,i)} : \ (a, i) \in \mathbb{Z}]$, where $\pi_{(a,i)} = P(Z_{(1)} = (a, i))$. The transition matrix of Z is denoted by a $dr \times dr$ matrix $G = [g_{ab}(ij) : \ (a, i), (b, j) \in \mathbb{Z}]$, where $g_{ab}(ij) = P(Z_{(k)} = (b, j) \mid Z_{(k-1)} = (a, i))$. The parameter of HBMM can be specified by $\phi = (\theta, G)$, and the total number of elements in ϕ is given by $L = 2d + d^2 r^2$, so we may also write $\phi = [\phi_\ell : \ell = 1, \ldots, L]$. We note that when $r = 1$, the HBMM reduces to the traditional HMM, where the state sojourn time distributions are geometric.

C. Online parameter estimation for HBMM

In [4], an online parameter estimation algorithm for the HBMM was developed, based on a block-recursive parameter estimation algorithm developed by Rydén [3] and a recursive

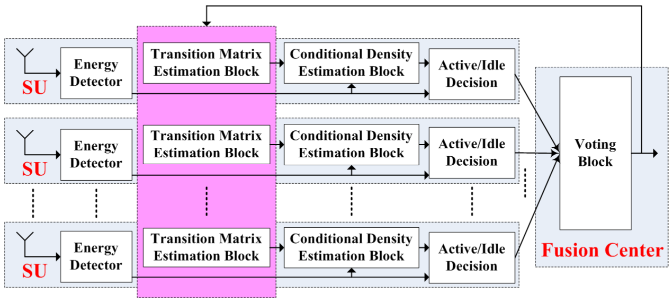

Fig. 1. Hard decision scheme.

score function algorithm from Willy [8]. This algorithm operates on a block of observation data at a time. Let m denote the block size, $y^m = \{y_1, \ldots, y_m\}$ denote a block of m observation samples, and let $p_\phi(y^m)$ denote the joint density of the observation block y^m. Let ϕ_n denote the nth HBMM parameter estimate computed by the online algorithm. The recursive algorithm can be expressed in the following form:

$$\phi_{n+1} = \Pi_{\mathcal{G}} \left[\phi_n + \gamma_n \chi \left(y^m ; \phi_n \right) \right], \tag{3}$$

where $\{\gamma_n\}$ is a coefficient sequence, $\Pi_{\mathcal{G}}$ denotes a projection operator mapping the estimate into a compact, convex set $\mathcal{G} \subseteq \Phi$, and $\chi(y^m; \phi)$ is the score function for the observation block y^m, given as follows:

$$\begin{aligned} \chi(y^m; \phi) &= \frac{\partial \log p_\phi(y^m)}{\partial \phi} \\ &= \sum_{(b,j)\in\mathbb{Z}} \frac{1}{p_\phi(y^m)} \frac{\partial}{\partial \phi} p_\phi(y^m, z_m = (b, j)). \end{aligned} \tag{4}$$

Under some mild assumptions, Rydén proved that for HMMs, the sequence $\{\phi_n\}$ converges to a point lying in the set of Kuhn-Tucker points for minimizing the Kullback-Leibler divergence defined over $\mathcal{G}$. As discussed in [4], Rydén's convergence results carry over to the HBMM.

The score function in (4) can be computed recursively as follows in terms of a $dr \times L$ matrix $H_m(y^m; \phi)$ whose (v, ℓ) element is given by

$$[H_m(y^m); \phi)]_{v\ell} = \frac{1}{p_\phi(y^m)} \frac{\partial}{\partial \phi_\ell} p_\phi(y^m, z_m = (b, j)), \tag{5}$$

where $(b, j) \in \mathbb{Z}$ such that $v = b(r-1) + j$ and $l \in L$. The matrix $H_m(y^m, \phi)$ can be computed recursively, as will be discussed below. Then the score function can be obtained from

$$\chi(y^m; \phi) = \mathbf{1}' H_m(y^m; \phi), \tag{6}$$

where $\mathbf{1}$ denotes a column vector of all ones and $'$ denotes matrix transpose.

III. Collaborative Sensing Schemes

A. Hard decision scheme

In a traditional hard decision fusion scheme, each SU makes a local decision on the state of the PU, and sends the binary decision (i.e., a single bit) to a fusion center. The fusion center makes a final decision using a fusion rule, e.g., the majority vote. Obviously, the main advantage of the traditional hard decision scheme is its low bandwidth requirement for the transmission of the single bit decision by each SU. However, the local decision made by an individual SU may not always be reliable, for example, when the received signal at SU experiences severe shadowing.

Our proposed hard decision fusion scheme is depicted in Fig. 1. In this scheme, the online HBMM parameter estimation algorithm of Section II-C is decoupled into two separate blocks: a transition matrix estimation block and a conditional density estimation block. The conditional density estimation block computes estimates for the mean and standard deviation of the conditional density for the Gaussian variable in (2). The transition matrix estimation block computes estimates of the transition matrix of the HBMM based on final decisions fed back to each SU from the fusion center. The transition matrix estimates are provided as input to the conditional density estimation block. The transition matrix estimates are updated less frequently than the sensing decisions, so a small delay in the feedback loop from the fusion center to the SUs will not adversely affect the overall performance of the scheme. The rationale for the separation of the transition matrix and output distribution parameter estimation is due to the fact that estimation of the parameter of the output distribution is significantly easier than that of the transition matrix.

We consider first the conditional density estimation block for the qth SU. To simplify notation, we shall drop the superscript (q) in the following discussion. We shall also reuse the notation ϕ to denote the parameter of interest for the conditional estimation block, given by $\phi = (\theta_a : a \in \mathbb{X})$. This parameter can be estimated in a block-recursive manner using (3), where the score function is a $1 \times 2d$ row vector given by (6), and $H_m(y^m; \phi)$ can be computed recursively as follows (cf. [4]):

$$\begin{aligned} H_m(y^m; \phi) = \frac{1}{c_m} \Big\{ & F(y_m)' H_{m-1}(y^{m-1}; \phi) \\ & + (I \otimes \xi_{m-1}) \frac{\partial}{\partial \phi} [\text{vec } F(y_m)]' \Big\}, \end{aligned} \tag{7}$$

where I denotes an identity matrix of order dr, $\otimes$ denotes the Kronecker product, the $dr \times dr$ matrix $F(y_m)$ is given by

$$F(y_m) = [f_{ij}^{ab}(y_m; \theta_b) : (a, i), (b, j) \in \mathbb{Z}], \tag{8}$$

with $f_{ij}^{ab}(y_m; \theta_b) \triangleq g_{ab}(ij) f(y_m; \theta_b)$, and vec $F(y_m)$ in (7) denotes the $d^2 r^2 \times 1$ column vector obtained by stacking the columns of the matrix $F(y_m)$ one on top of the other. The elements of the $d^2 r^2 \times 2d$ Jacobian matrix $\partial [\text{vec } F(y_m)]' / \partial \phi$ are partial derivatives of $f_{ij}^{ab}(y_m; \theta_b)$, given as follows:

$$\begin{aligned} \frac{\partial f_{ij}^{ab}(y_m; \theta_b)}{\partial \mu_c} &= f_{ij}^{ab}(y_m; \theta_b) \cdot \frac{y_m - \mu_b}{\sigma_b^2} \mathbf{1}_{\{c=b\}}, \\ \frac{\partial f_{ij}^{ab}(y_m; \theta_b)}{\partial \sigma_c} &= f_{ij}^{ab}(y_m; \theta_b) \cdot \frac{(y_m - \mu_b)^2 - \sigma_b^2}{\sigma_b^3} \mathbf{1}_{\{c=b\}}, \end{aligned} \tag{9}$$

for $c \in \mathbb{X}$, where $\mathbf{1}_A$ denotes an indicator function on set A. The $1 \times dr$ row vector ξ_m and scalar c_m are given by

$$\xi_m = \frac{1}{c_m}\xi_{m-1}F(y_m), \quad c_m = \xi_{m-1}F(y_m)\mathbf{1}, \tag{10}$$

for $m = 1, 2, \ldots$, with the initial condition $\xi_0 = \pi$.

The transition matrix estimation block at each SU can be parametrized by $\phi = [g_{ab}(ij) : \ (a,i),(b,j) \in \mathbb{Z}]$. Since the observation data comes from the final decisions of the fusion center, the observable process Y takes values in the state-space $\mathbb{X}$, i.e., the HBMM reduces to bivariate Markov chain. In this case, the parameter ϕ can also be estimated by using (3), (4), and (7), but with $f_{ij}^{ab}(y_m;\theta_b)$ in (8) given as follows:

$$f_{ij}^{ab}(y_m;\theta_b) \triangleq g_{ab}(ij)f(y_m;\theta_b) = g_{ab}(ij)\ \mathbf{1}_{\{y_m=b\}}. \tag{11}$$

In addition, the elements of the $d^2r^2 \times d^2r^2$ Jacobian matrix $\partial[\text{vec } F(y_{(m)})]'/\partial\phi$ for (7) are given as follows:

$$\frac{\partial f_{ij}^{ab}(y_m;\theta_b)}{\partial[g_{ce}(\iota l)]} = \mathbf{1}_{\{y_m=b,(c,\iota)=(a,i),(e,l)=(b,j)\}}, \tag{12}$$

for $(c,\iota),(e,l) \in \mathbb{Z}$.

Each SU makes a local decisions based on the observation data and the estimated conditional density. The mean and standard deviation for the conditional density are updated after each new block of observation data. The state detection scheme for each SU is given by

$$\hat{X}_k = \begin{cases} 1, & \text{if } Y_k \le \gamma_h, \\ 2, & \text{otherwise}, \end{cases} \tag{13}$$

where γ_h is the corresponding decision threshold. For each SU, the detection probability P_d can be expressed in terms of the false alarm probability P_{fa} as follows [7]:

$$P_d = Q\left[\frac{\sigma_1 Q^{-1}(P_{fa}) + \mu_1 - \mu_2}{\sigma_2}\right], \tag{14}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{u^2}{2}}du$ denotes the standard Q-function and the decision threshold is given by

$$\gamma_h = \mu_1 + \sigma_1 Q^{-1}(P_{fa}). \tag{15}$$

The local SU decisions are sent to the fusion center, which makes a final decision on the state of the PU at time k via a majority voting rule.

B. Soft decision scheme

In the soft decision scheme shown in Fig. 2, the received signal strength from the output of the front-end at each SU is transmitted to the fusion center in each time slot. High resolution quantization of the data is assumed and hence quantization noise is ignored here but will be taken into account in a forthcoming study. Thus, the soft decision scheme has a much higher communication overhead than the hard decision scheme, but the fusion center has the potential of achieving better detection performance given the observable data from all of the SUs. We adopt the linear combining approach discussed in [7], which involves assigning a weight $w_k^{(q)}$ at time k to each SU q, $q = 1, \ldots, Q$. Define a row vector $\boldsymbol{w}_k = \{w_k^{(q)} : 1, \ldots, Q\}$. The soft decision variable is given by the weighted sum of the observed samples from the SUs at time k: $V_k = \boldsymbol{w}_k\boldsymbol{Y}_k'$.

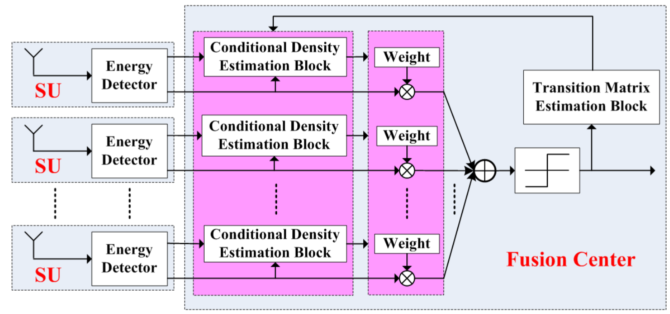

Fig. 2. Soft decision scheme.

In [7], a heuristic method is proposed to assign the weights, under the assumption that the parameters of the wireless channels for each of the SUs are known. In the soft decision scheme of Fig. 2, the channel parameters are estimated as part of the procedure for estimating the HBMM. In the soft decision scheme, only one transition matrix estimation block is needed because all SUs perform spectrum sensing for the same PU. A similar argument could be made for the hard decision scheme, but in Fig. 1 a separate transition matrix estimation block is assigned to each SU to avoid the need for the fusion center to send transition matrix estimates to each SU, which would incur significant communication overhead. In Fig. 2, the qth conditional density estimation block computes an estimate of $\theta^{(q)} = [(\mu_a^{(q)}, \sigma_a^{(q)}) : a = 1, 2]$. Let $\boldsymbol{\mu}_a = (\mu_a^{(q)} : q = 1, \ldots, Q)$ denote a row vector of mean signal strengths received by the SUs when the PU is in state a. Similarly, let $\Sigma_a = \text{diag}([\sigma_a^{(q)}]^2 : q = 1, \ldots, Q)$ denote the covariance matrix for the received signal strengths received by the SUs when the PU is in state a.

The soft decision scheme is given by

$$\hat{X}_k = \begin{cases} 1, & \text{if } \boldsymbol{w}_k\boldsymbol{Y}_k' \le \gamma_s, \\ 2, & \text{otherwise}, \end{cases} \tag{16}$$

where the sequence of weight vectors $\{\boldsymbol{w}_k\}$ and the threshold γ_s are determined from the estimated HBMM parameter as shown below. Since the variables in $\boldsymbol{Y}_k$ are Gaussian, the soft decision variable V_k is conditionally Gaussian with conditional mean and variance given, respectively, as follows:

$$\text{E}[V_k|X_k = a] = \boldsymbol{w}_k\boldsymbol{\mu}_a', \ \text{Var}[V_k|X_k = a] = \boldsymbol{w}_k\Sigma_a\boldsymbol{w}_k'.$$

The detection probability P_d can be expressed in terms of the false alarm probability P_{fa} as follows:

$$P_d = Q\left[\frac{Q^{-1}(P_{fa})\sqrt{\boldsymbol{w}_k\Sigma_1\boldsymbol{w}_k'} - \boldsymbol{w}_k\boldsymbol{\mu}'}{\sqrt{\boldsymbol{w}_k\Sigma_2\boldsymbol{w}_k'}}\right], \tag{17}$$

where $\boldsymbol{\mu} \triangleq \boldsymbol{\mu}_2 - \boldsymbol{\mu}_1$ and the threshold is given by

$$\gamma_s = \boldsymbol{w}_k\boldsymbol{\mu}_1' + Q^{-1}(P_{fa})\sqrt{\boldsymbol{w}_k\Sigma_1\boldsymbol{w}_k'}. \tag{18}$$

It remains to compute the weight vector $\boldsymbol{w}_k$. The modified deflection coefficient (MDC) defined in [7] is given as follows in our soft collaboration scheme:

$$\mathrm{MDC}(\boldsymbol{w}_k) = \frac{(\mathrm{E}[V_k|X_k=2] - \mathrm{E}[V_k|X_k=1])^2}{\mathrm{Var}[V_k \mid X_k=2]} = \frac{(\boldsymbol{w}_k\boldsymbol{\mu}')^2}{\boldsymbol{w}_k\Sigma_2\boldsymbol{w}_k'}.$$

The MDC is maximized under the unit-norm constraint on the weight vector, i.e.,

$$\begin{aligned} &\max\ \mathrm{MDC}(\boldsymbol{w}_k) \\ &\text{s.t. } \|\boldsymbol{w}_k\|_2^2 = 1, \end{aligned} \tag{19}$$

where $\|\cdot\|_2$ denotes the standard Euclidean norm. The optimal solution for (19) in our collaboration model can be derived as in [7] and is given by

$$\boldsymbol{w}_k = \frac{\Sigma_2^{-1}\boldsymbol{\mu}'}{\|\Sigma_2^{-1}\boldsymbol{\mu}'\|_2}. \tag{20}$$

IV. Numerical Results

We have run simulation experiments in MATLAB to evaluate the performance of the proposed hard and soft decision spectrum sensing schemes. For the online parameter estimation algorithm given in (3), we set $\gamma_n = \gamma_0 n^{-\varepsilon}$ with $\gamma_0 = 0.3$ and $\varepsilon = 0.35$. We set the block size $m = 20$. The parameter space $\mathcal{G}$ and projection operator $\Pi_{\mathcal{G}}$ in (3) are the same in [4]. To improve the convergence speed and stability, a warmup period is introduced to provide a good initial estimate for the mean and standard deviation vectors associated with the conditional density estimation block in both the hard and soft decision schemes. Given a sequence y^{n_0} of length $n_0 = 200$, we use the following initialization procedure:

1) Let $\mathbb{A}_1 = \{k \in \{1,\ldots,n_0\} : y_k < (\max(y^{n_0}) + \min(y^{n_0}))/2\}$ and $\mathbb{A}_2 = \{1,\ldots,n_0\}\backslash\mathbb{A}_1$.
2) The initial estimates $\theta_a^s = (\mu_a^s, \sigma_a^s)$ are computed as follows:

$$\mu_a^s = \frac{1}{|\mathbb{A}_a|}\sum_{k\in\mathbb{A}_a} y_k, \quad \sigma_a^s = \sqrt{\frac{1}{|\mathbb{A}_a|-1}\sum_{k\in\mathbb{A}_a}|y_k-\mu_a^s|^2}, \tag{21}$$

where $|\cdot|$ denotes set cardinality and $a \in \mathbb{X}$.

The initial probability vector π^s is initialized with a uniform distribution and the initial transition matrix G^s is generated randomly.

In our simulation experiments, we considered a collaboration model with three SUs (i.e., $Q = 3$). For the true parameter ϕ^0, the state transition matrix G^0 is specified by a 20×20 transition matrix adopted from [9], such that $d = 2$ and $r = 10$. The true transition matrix was estimated from real spectrum measurements of a paging channel collected in [10] using the Baum algorithm. For the estimation blocks we have set $d = 2$ and $r = 10$, but a smaller value of r could be used to trade off accuracy for a reduction in computational complexity. The true mean and standard deviations of the conditional densities for three SUs are given in the Table I. Two different scenarios representing different channel condition for three SUs are shown in this table.

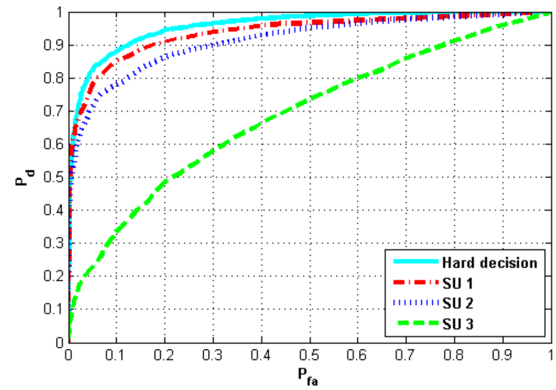

Fig. 3. ROC plot of hard decision scheme in first scenario.

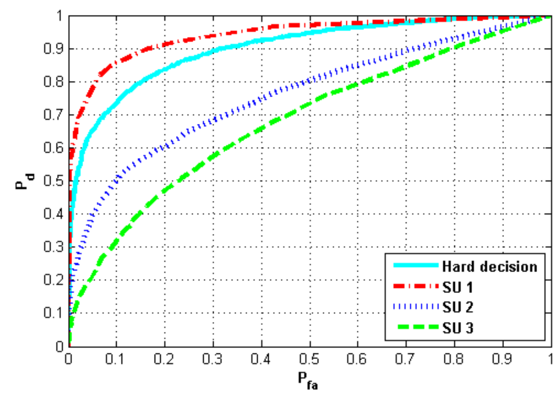

Fig. 4. ROC plot of hard decision scheme in second scenario.

We evaluated the detection performance of the hard decision scheme by applying $T = 6000$ observation samples to obtain an HBMM estimate for each SU. Each receiver operating characteristic (ROC) curve in Figs. 3 and 4 was then obtained by applying the state estimator in (13) with $10,000$ new observation samples. The ROC performance for the first scenario is shown in Fig. 3. From Fig. 3 we find that SU1 and SU2 have better performance compared to SU3, and the hard decision scheme can achieve superior performance compared to that of the individual SUs. The ROC performance for the second scenario is shown in Fig. 4. From Fig. 4, we find that the performance of SU2 and SU3 are worse compared to the performance of SU1, and the performance of hard decision scheme is inferior to the performance of SU1. The result agrees with the intuition that hard decision fusion works well when the majority of SUs have good performance; otherwise, the performance may be degraded.

We also carried out similar simulation experiments with the soft decision scheme. First, HBMM parameter estimates for each SU were obtained by applying T observation samples to the soft decision scheme of Fig. 2, with $T = 200, 600, 6000$. Then each ROC curve was obtained using the state estimator in (16) by applying 10,000 additional observation samples. The ROC performance for the first scenario is shown in Fig. 5.

	$(\mu_1^{(1)},\sigma_1^{(1)})$	$(\mu_2^{(1)},\sigma_2^{(1)})$	$(\mu_1^{(2)},\sigma_1^{(2)})$	$(\mu_2^{(2)},\sigma_2^{(2)})$	$(\mu_1^{(3)},\sigma_1^{(3)})$	$(\mu_2^{(3)},\sigma_2^{(3)})$
Scenario 1	(-105.00,6.32)	(-88.00,8.94)	(-100.00,5.91)	(-86.00,8.36)	(-103.00,6.71)	(-98.00,8.06)
Scenario 2	(-105.00,6.32)	(-88.00,8.94)	(-102.00,5.91)	(-95.00,8.36)	(-103.00,6.71)	(-98.00,8.06)

TABLE I
TRUE MEAN AND STANDARD DEVIATION OF CONDITIONAL DENSITIES.

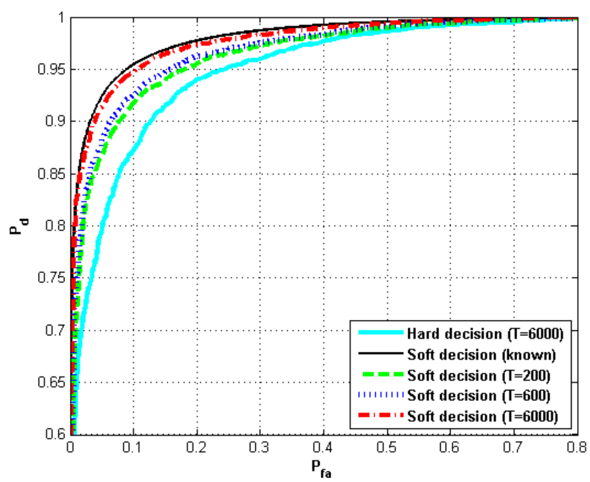

Fig. 5. ROC plot of soft decision scheme in first scenario.

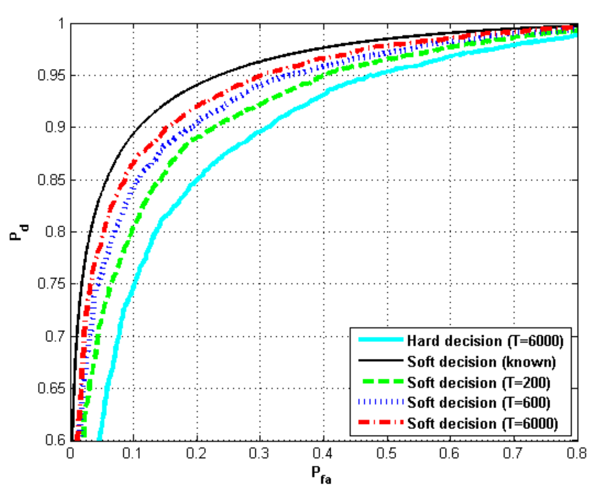

Fig. 6. ROC plot of soft decision scheme in second scenario.

From Fig. 5, we find that the ROC performance of the soft decision scheme improves when the observation data length for estimating the HBMM parameter increases from $T = 200$ to $T = 6000$ and eventually approaches the performance of the soft decision scheme when the wireless channel parameters are known. We also find that the ROC performance for the soft decision scheme is better than that of the hard decision scheme. A similar result for the second scenario is shown in Fig. 6.

V. CONCLUSION

We have developed hard and soft decision schemes for collaborative spectrum sensing of a cognitive radio channel based on online parameter estimation of a hidden bivariate Markov model. The advantage of the proposed approach lies in its ability to perform well under low signal-to-noise ratio conditions due to the statistical characterization of the channel and primary user behavior provided by the model. The online parameter estimation scheme allows the values of the hard decision thresholds and the soft decision weights and thresholds to be adapted dynamically in accordance with changes in the wireless environment.

In ongoing work, we are investigating extensions to the proposed model-based collaborative sensing schemes. First, the soft linear combiner at the fusion center can be replaced by a nonlinear combiner based on state estimation of the hidden bivariate Markov model. Such a scheme should achieve superior detection performance relative to the linear scheme. Second, the model-based collaborative sensing approach opens the door for predictive collaborative sensing, wherein future states of the primary user are predicted using the parameter estimate for the hidden bivariate Markov model. Such predictive spectrum sensing could lead to more efficient spectrum utilization and reduced interference to primary users.

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