An Analytical Performance Model of Opportunistic Spectrum Access in a Military Environment

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Abstract— **In an opportunistic spectrum sharing system, secondary users equipped with cognitive radios opportunistically access spectrum that is not being used by the primary users, i.e., the licensed spectrum users, without causing harmful interference to the primary users. We present an analytical performance model of opportunistic spectrum access in a military environment consisting of a group of secondary users sharing a set of channels with primary users in a coverage area. A secondary user occupying a given channel detects when a primary user accesses the channel and then either moves to another idle channel or is placed in a virtual queue where it waits until either a channel becomes available or a maximum waiting time is reached. Using a two-dimensional Markov model, we derive expressions for the blocking probabilities and reconnection probability and evaluate the performance metrics under a range of parameter settings.**

I. Introduction

Opportunistic spectrum sharing (OSS) using cognitive radios [**?**] is a promising approach to reusing allocated radio spectrum. The IEEE 802.22 standard defines a cognitive radio as a radio transmitter/receiver designed to intelligently detect whether a particular segment of the radio spectrum is currently in use and to jump into (or out of) the temporarily-unused spectrum very rapidly without interfering with the transmissions of other users. Cognitive radios offer a great number of benefits in commercial, government, and especially military applications. The U.S. Department of Defense (DoD) has established programs such as Joint Tactical Radio System (JTRS) and neXt Generation (XG) to further explore cognitive radio technology.

In a military scenario, troops deployed on foreign soil communicate with each other opportunistically when spectrum becomes available, without causing harmful interference to the authorized or *primary* users of the spectrum. We refer to the unauthorized users as the *secondary* users. The wireless system consisting of both primary and secondary users is an example of an OSS system. The subsystem consisting only of the primary users is called the *primary system*, while the subsystem consisting only of the secondary users is called the *secondary system*.

This paper presents an analytical model to evaluate the performance of an OSS system in a military environment.[1] A key characteristic of the military OSS system is that the set of secondary users is of a fixed size, whereas the set of primary users is large enough to be modeled accurately as an infinite source. The primary system may or may not be infrastructured. However, the secondary system operates in ad hoc or infrastructureless mode. An OSS with an infrastructured primary system is depicted in Fig. 1. The allocated spectrum is divided into a set of channels. Here, we use the term channel in a broad sense. A channel could be a frequency channel in an FDMA system, a time-slot in a TDMA system, a spreading code in a CDMA system, or a tone in an OFDM system. The system model considered in this paper can be applied to all of these scenarios.

Secondary users opportunistically make use of channels that are not occupied by primary users. We assume that each secondary user is equipped with a cognitive radio, which is capable of sensing when a channel is idle and then making use of the channel. Secondary users may collaborate with each other to identify the status of a channel. A secondary user can also detect when a primary user accesses a channel that it is using and then either move to another channel, if an idle channel is available, or move to a waiting pool. In the latter case, the secondary user's call waits in a virtual queue until either a new channel becomes available or until a timeout occurs after a predefined maximum waiting time. The virtual queue can be implemented in a distributed manner among the secondary users, or one of the secondary users can assume the role of managing the virtual queue.

The reliable detection of primary user spectrum occupancy is a major challenge for the implementation of an OSS system. The spectrum usage of the secondary users is contingent on the requirement that the interference to the primary users must be limited to a certain threshold. A number of opportunistic spectrum access (OSA) schemes have been developed recently in the literature [**?**], [**?**], [**?**], [**?**]. In [?], a framework was developed for modeling the interference caused by cognitive employing spectrum access mechanisms based on the Listen-Before-Talk (LBT) scheme. Two variations of LBT were considered: individual LBT and collaborative LBT. In [?], collaborative spectrum sensing was proposed and studied as a means to combat the shadowing or fading effects that a user experiences. In [?], the authors showed that, by taking advantage of the local oscillator leakage power that all RF receivers emit, sensor nodes could detect the exact channel that a primary user was tuned to and transmit this information to the cognitive radios. In [?], a measurement-based model was proposed to statistically characterize the busy and idle periods of a WLAN. Two sensing strategies,

[1]This work was supported in part by the National Science Foundation under Grants CNS-0520151 and ACI-0133390.

energy-based detection and feature-based detection, were explored to identify spectrum opportunities.

In this paper, we focus the performance analysis of an OSS system model at the call level under the assumption of perfect OSA, i.e., the secondary users are able to move in and out of channels to avoid harmful interference with primary users. We evaluate the performance of a generic OSS system in a military environment in terms of blocking probabilities of primary and secondary users and reconnection probability of secondary users. In an earlier paper [**?**], we presented a similar performance analysis of an OSS system in which the secondary system is modeled by an infinite population source. Such a model is not appropriate for military environment where the secondary users form an infrastructureless network with a small population.

The remainder of the paper is organized as follows. Section II describes the system model and assumptions in further detail. Section III develops a Markovian model of the system dynamics and derives the performance metrics of interest. Section IV presents numerical results, illustrating the performance of the OSS system with respect to the different metrics, over a range of parameter settings. Finally, the paper is concluded in Section V.

II. Model Description

Consider a cellular network operating over a given geographical area to serve primary users. The network owns the license for spectrum usage in the service area. Calls generated by primary users constitute the primary traffic (PT) stream. Each cell in the primary system contains a base station (BS), which has wireline connectivity to the backbone network. A primary user within a cell sets up a call through the BS to communicate with other primary users. Although we assume in this paper that the primary system is a cellular network, primary system could represent other infrastructured or infrastructureless wireless systems, such as a wireless LAN, a WiMAX system, a wireless ad hoc network, etc.

A group of secondary users in the same service area, which opportunistically access the spectrum resources to communicate with each other in ad hoc or infrastructureless mode. In a typical military scenario, the secondary system is infrastructureless and has a small population. Calls generated by secondary users constitute the secondary traffic (ST) stream. The system consisting of the primary and secondary systems is called an opportunistic spectrum sharing (OSS) system. In the OSS system, the spectrum availability for the secondary users depends on the spectrum occupancy of the primary users. A distinct feature of the OSS system is that the secondary users have the capability to sense channel usage and switch between different channels using appropriate communication mechanisms, while causing negligible interference to the primary users. Such functionality can be realized by cognitive radios, as mentioned earlier.

An opportunistic spectrum sharing (OSS) system model is depicted in Fig. 1. Assume that there are a total of N channels in a given cell. The PT calls operate as if there are no ST calls in the system. When a PT call arrives to the system, it occupies a free channel if one is available; otherwise, it will be blocked. Secondary users detect the presence or absence of signals from primary users and maintain records of the channel occupancy status. The detection mechanism may involve collaboration with other secondary users.

Secondary users opportunistically access the channels that are in idle status. When an ST node in communication detects or is informed by other ST nodes of the arrival of a PT call in its current channel, it immediately leaves the channel and switches to an idle channel, if one is available, to continue the call (see Fig. 1). If at that time all the channels are occupied, the ST call is placed into a virtual queue. For example, in Fig. 1, when an ST call detects the arrival of a PT call at channel i, it immediately leaves that channel and moves to channel j. If all of the N channels are occupied at that time, the ST call enters the virtual queue. The virtual queue may be implemented in a distributed manner by the group of secondary users, or alternatively, one of the secondary users can take responsibility for managing the virtual queue.

Queued ST calls are served in first-come first-served (FCFS) order. The head-of-line ST call is reconnected to the system when a channel becomes available before a predefined maximum waiting time expires. We set the maximum waiting time of an ST call equal to its residence time in the considered service area. Thus, an ST call is lost only when it moves out of the service area, statistically speaking. A similar model was developed in [**?**] to analyze multi-service mobile cellular networks, where real-time calls have preemptive priority over non-real-time calls. The model in the present paper differs from the model of [?] in that the secondary users cannot communicate with primary users to optimize the management of the shared resource and the secondary users have a limited number of sources.

III. Performance Analysis

In this section, we analyze the OSS system performance in a given service area consisting of the primary and secondary systems sharing the same spectrum. The spectrum is divided into N channels serving the two types of traffic: primary traffic (PT) and secondary traffic (ST). Assume that the arrival process of the PT calls is a Poisson process with rate λ. The channel holding time of a PT call is exponentially distributed with mean $1/\mu_1$. In contrast to the infinite source model for PT calls, the ST calls are generated from a finite source model consisting of K secondary users in the OSS system. In our analysis, the following two cases are considered separately: (i) the secondary user population is less than the total number of channels, i.e., $K < N$; (ii) $K \geq N$.

The individual ST call switches between the sensing state and communication state. We shall assume that an individual ST call senses for an exponentially

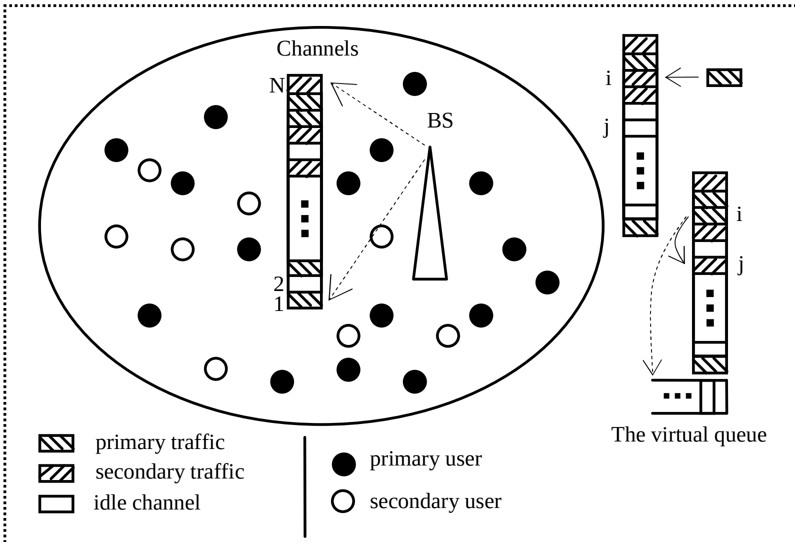

Fig. 1. An opportunistic spectrum sharing (OSS) system.

distributed time interval with parameter ν, and the ST call communicates for an exponentially distributed time interval (i.e., the channel holding time) with parameter μ_2. The residence time of the ST calls in the considered service area is assumed to be exponentially distributed with parameter r_2. According to our assumption in the previous section, the maximum waiting time of an ST call in the virtual queue has the same distribution as the residence time; hence, the maximum waiting time of an ST call is exponentially distributed with parameter r_2. We further assume that each type of traffic occupies one channel for simplicity. However, the analysis method used here can be extended to handle bandwidth requests of variable size (cf. [**?**]).

Let $X_1(t)$ denote the number of PT calls in the OSS system at time t. Similarly, let $X_2(t)$ be the number of ST calls in the system at time t, including the ST calls being served and those waiting in the virtual queue. The process $(X_1(t), X_2(t))$ is a two-dimensional Markov process with state space

$$\mathcal{S} = \{(n_1, n_2) | 0 \le n_1 \le N,\ 0 \le n_2 \le N^*\},$$

where $N^* \triangleq \min(K, N)$. We denote the transition rate from state (n_1, n_2) to (n_1', n_2') by $T_{n_1,n_2}^{n_1',n_2'}$ and specify the nonzero transition rates as follows.

$$\begin{aligned} T_{n_1,n_2}^{n_1+1,n_2} &= \lambda, & n_1 < N, n_2 \le N^*, \\ T_{n_1,n_2}^{n_1-1,n_2} &= n_1\mu_1, & 1 \le n_1 \le N, n_2 \le N^*. \end{aligned}$$

The transition rates from (n_1, n_2) to (n_1, n_2+1) and to (n_1, n_2-1) depend on the relationship between K and N. When $K \ge N$,

$$T_{n_1,n_2}^{n_1,n_2+1} = (K - n_2)\nu,\ \ n_1 \le N-1, n_2 < N - n_1,$$

$$T_{n_1,n_2}^{n_1,n_2-1} = \begin{cases} n_2\mu_2, \quad n_1 \le N-1, \\ \qquad\quad 1 \le n_2 \le N - n_1, \\ (N-n_1)\mu_2 + (n_2 - N + n_1)r_2, \\ \qquad\quad 1 \le n_1 \le N, N - n_1 < n_2 \le N. \end{cases}$$

When $K < N$,

$$T_{n_1,n_2}^{n_1,n_2+1} = (K - n_2)\nu,$$

for $n_1 \le N-K, n_2 < K-1$ and for $N-K+1 \le n_1 \le N-1, n_2 < N-n_1-1$. Finally, $T_{n_1,n_2}^{n_1,n_2-1} =$

$$\begin{cases} n_2\mu_2, \quad n_1 \le N-K, 1 \le n_2 \le K, \\ \qquad\quad N-K+1 \le n_1 \le N, 1 \le n_2 \le N - n_1), \\ (N-n_1)\mu_2 + (n_2 - N + n_1)r_2, \\ \qquad\quad N-K+1 \le n_1 \le N, N-n_1+1 \le n_2 \le K. \end{cases}$$

Let $\pi(n_1, n_2)$ denote the steady-state probability that the OSS system is in state (n_1, n_2). The steady-state system probability vector, with states ordered lexicographically, can be represented as

$$\boldsymbol{\pi} = (\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \cdots, \boldsymbol{\pi}_N),$$

where

$$\boldsymbol{\pi}_n = (\pi(n, 0), \pi(n, 1), \cdots, \pi(n, N^*)),\ \ 0 \le n \le N.$$

The vector $\boldsymbol{\pi}$ is the solution of

$$\boldsymbol{\pi} Q = \mathbf{0} \text{ and } \boldsymbol{\pi}\mathbf{e} = \mathbf{1},$$

where $\mathbf{e}$ and $\mathbf{0}$ are column vectors of all ones and zeros, respectively. The infinitesimal generator, Q, of the two-dimensional Markov process, is given by

$$Q = \begin{bmatrix} E_0 & B_0 & 0 & \cdots & 0 & 0 & 0 \\ D_1 & E_1 & B_1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & D_{N-1} & E_{N-1} & B_{N-1} \\ 0 & 0 & 0 & \cdots & 0 & D_N & E_N \end{bmatrix}$$

where each submatrix is of size N^*+1 by N^*+1 and defined by

$$\begin{aligned} B_i &= \lambda I_{N^*+1}, \quad 0 \le i < N, \\ D_i &= i\mu_1 I_{N^*+1}, \quad 1 \le i \le N, \\ E_i &= A_i - \bar{\delta}(i) D_i - \bar{\delta}(N-i) B_i, \quad 0 \le i \le N, \end{aligned}$$

where I_n denotes an n-by-n identity matrix, $\bar{\delta}(i)$ is 0 when $i = 0$ and 1 otherwise. The matrix A_i has the same size as E_i. Let $A_i(j,k)$ denote the (j,k)-element of A_i. Define

$$A_i(j,k) = 0, \quad \text{for } |j-k| > 1.$$

The nonzero elements of A_i are specified below. For $K \geq N$:

$$A_i(j,j{+}1) = (K-j)\nu, \quad i \leq N{-}1, j \leq N{-}i{-}1,$$

$$A_i(j,j{-}1) =$$
$$\begin{cases} j\mu_2, & i \leq N{-}1, 1 \leq j \leq N{-}i, \\ (N-i)\mu_2 + (j{-}N{+}i)r_2, & 1 \leq i \leq N, \\ & N{-}i{+}1 \leq j \leq N, \end{cases}$$

$$A_i(j,j) = -[A_i(j,j{-}1) + A_i(j,j{+}1)], \; i,j \leq N.$$

For $K < N$:

$$A_i(j,j{+}1) = (K-j)\nu, \quad i \leq N{-}K, j \leq K{-}1, \\ \text{or } N{-}K < i \leq N{-}1, j < N{-}i.$$

$$A_i(j,j{-}1) =$$
$$\begin{cases} j\mu_2, & i \leq N{-}K, 1 \leq j \leq K, \\ & \text{or } N{-}K < i \leq N{-}1, 1 \leq j \leq N{-}i, \\ (N-i)\mu_2 & \\ +(j{-}N{+}i)r_2, & N{-}K < i \leq N, N{-}i < j \leq K, \end{cases}$$

$$A_i(j,j) = -[A_i(j,j-1) + A_i(j,j+1)],$$

for $i \leq N, j \leq K$.

Applying the method developed in [?], the equilibrium state probabilities can be determined as

$$\boldsymbol{\pi}_n = \boldsymbol{\pi}_{n-1} B_{n-1} \left(-C_n\right)^{-1} = \boldsymbol{\pi}_0 \prod_{i=1}^{n} [B_{i-1} \left(-C_i\right)^{-1}],$$

where $1 \leq n \leq N$, $\boldsymbol{\pi}_0$ satisfies $\boldsymbol{\pi}_0 C_0 = \mathbf{0}$, and

$$\boldsymbol{\pi}_0 \left[I + \sum_{n=1}^{N} \prod_{i=1}^{n} [B_{i-1} \left(-C_i\right)^{-1}] \right] \mathbf{e} = 1.$$

The C_i are computed recursively as follows:

$$C_N = E_N,$$
$$C_i = E_i + B_i \left(-C_{i+1}\right)^{-1} D_{i+1}, \; 0 \leq i \leq N-1.$$

Given the steady state probabilities, we can then determine various performance measures of interest.

A. PT Call Blocking Probability

The PT call blocking probability, denoted by P_1, is defined as the probability that upon an arrival of a PT call in a service area all the channels are occupied by PT calls and the arrival request has to be blocked. Thus, we have

$$P_1 = \sum_{n_2=0}^{N^*} \pi(N, n_2) = \boldsymbol{\pi}_0 \prod_{i=1}^{N} [B_{i-1} \left(-C_i\right)^{-1}] \mathbf{e}. \tag{1}$$

B. ST Call Blocking Probability

The ST call blocking probability, denoted by P_2, is defined as the probability that all the channels in a service area are occupied by either PT calls and/or ST calls and no channel is available for a new ST call request. When $K \geq N$, we have

$$P_2 = \sum_{n_1=0}^{N} \sum_{n_2=N-n_1}^{N} \pi(n_1, n_2). \tag{2}$$

When $K < N$, we have

$$P_2 = \sum_{n_1=N-K+1}^{N} \sum_{n_2=N-n_1}^{K} \pi(n_1, n_2). \tag{3}$$

When $n_2 = K$ in the above equation (3), P_2 can be interpreted as the blocking probability of an “imaginary” ST call request.

C. Mean Reconnection Probability

As mentioned earlier, an ST call that waits in the virtual queue due to unavailability of a channel could reconnect back to the system if a channel becomes available before the maximum waiting time expires. The mean reconnection probability of an ST call, denoted by γ, is defined as the probability that this ST call reconnects back to the system before its maximum waiting time expires. We can derive the following expression for γ:

$$\gamma = \frac{\sum_{n_1=\alpha}^{N} \sum_{j=0}^{n_1-\alpha} \pi(n_1, N-n_1+j+1)\beta(j)}{\sum_{n_1=\alpha}^{N} \sum_{j=0}^{n_1-\alpha} \pi(n_1, N-n_1+j+1)}, \tag{4}$$

where $\alpha \triangleq \max(1, N-K+1)$. Here, $\beta(j)$ denotes the probability that an ST call arriving at the queue eventually reconnects back to the system before its maximum queueing time expires, given that it finds that there are j ST calls in the queue ($0 \leq j \leq N^*-1$). When an ST call in the communication state detects the arrival of a PT call on its current channel, it switches to another idle channel if one is available; otherwise, the call is queued in the virtual queue.

The ST calls in the queue are reconnected to the system, when channels become available, in first come first served (FCFS) order. More precisely, if an ST call in the communication state detects an arrival of a PT call to its current channel and there are $N+j$ ($0 \leq j \leq N^*-1$) calls in the system (i.e., all N channels are serving the PT/ST calls and j ST calls are waiting in the queue), it releases its channel for the PT call and enters the queue, which leads to a new system state with all N channels being used and $j+1$ ST calls in the queue. This ST call reconnects to the system only if $j+1$ calls leave the service area (either releasing a channel or a position in the queue) before its maximum queueing time expires.

If we let τ denote the maximum queueing time that can be tolerated by an ST call in the queue, then τ is exponentially distributed with mean $1/r_2$. Let φ_j ($0 \leq j \leq N^*-1$) denote the time interval from the epoch that an ST call enters the queue with all N channels being occupied and j ST calls in the queue to the epoch at which one of the calls (either an ongoing PT/ST call or a queued ST call) leaves the system. More precisely, if the ST call that just released its channel arrives to find j ST calls waiting in the queue, n_1 PT calls and $N - n_1$ ST calls being served in the system ($1 \leq n_1 \leq N, 0 \leq j \leq N^* - 1$), then either a PT call's completion or an ST call's completion will subsequently result in the call at the head of the queue reconnecting back to the system. As a result, each of the remaining ST calls in the queue move forward by one position in the queue. Similarly, the event of a queued ST call leaving the system will result in each of the remaining queued ST calls behind it to move forward by one position. Thus, φ_j is exponentially distributed with rate $n_1\mu_1 + (N - n_1)\mu_2 + jr_2$.

Let $f_j(\cdot)$ denote the probability density function of φ_j and let $f_j^*(s)$ denote its Laplace transform. By the assumption of independence of the random variables φ_j, we can determine $\beta(j)$ as

$$\beta(j) = \Pr(\tau > \varphi_0 + \varphi_1 + \cdots + \varphi_j) = \prod_{i=0}^{j} f_i^*(r_2)$$
$$= \frac{n_1\mu_1 + (N - n_1)\mu_2}{n_1\mu_1 + (N - n_1)\mu_2 + (j+1)r_2}, \quad (5)$$

where the last equation follows from the fact that

$$f_i^*(r_2) = \frac{n_1\mu_1 + (N - n_1)\mu_2 + ir_2}{n_1\mu_1 + (N - n_1)\mu_2 + (i+1)r_2}.$$

The reconnection probability can then be calculated by substituting (5) into (4).

IV. Numerical Results

We present numerical results in terms of the blocking probabilities of PT and ST calls as well as the mean reconnection probability. We also compare the system performance under a small number ($K < N$) and a large number ($K \geq N$) of secondary users. The system parameter settings are as follows: $N = 16$, $\mu_1 = 10$, $\mu_2 = 20$, $r_2 = 10$. The arrival rate of the primary users is set to be $\lambda = 10 \sim 90$, and the arrival rate of the individual secondary user is $\nu = 10$. The number of secondary users is set to be $K = 10$ for a small group of secondary users and $K = 24$ for a large group.

Fig. 2 depicts the PT call blocking probability under different conditions. We observe that the PT call blocking probability does not change with respect to the arrival rate and number of the secondary users. This is because in our system model, the channel availability for an ST call depends on the requirement that it cannot interfere with a PT call. Thus, the PT calls are oblivious to the existence of ST calls under a perfect OSA mechanism.

Fig. 3 depicts the ST call blocking probability under different conditions. Observe that the ST call blocking probability increases with respect to the PT call arrival rate λ or the individual ST call arrival rate ν. This can be explained as follows: As λ increases, the number of available channels that can be opportunistically accessed

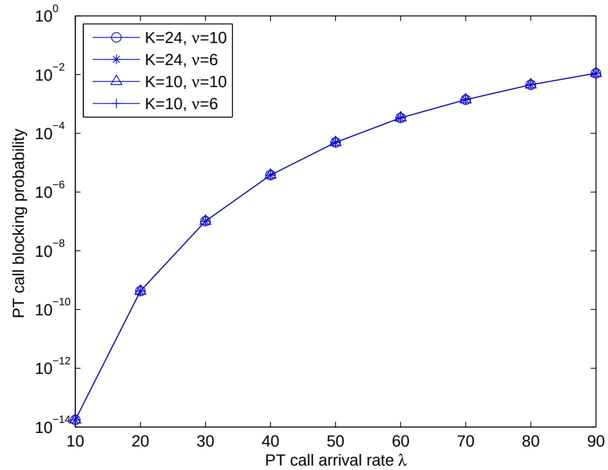

Fig. 2. The PT call blocking probability.

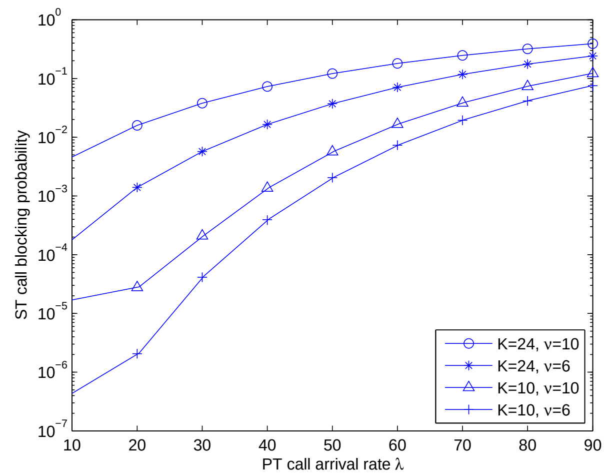

Fig. 3. The ST call blocking probability.

by the secondary users reduces, resulting in higher ST call blocking probability. As ν increases, a larger number of secondary users contends for the available channels, which similarly results in higher ST call blocking probability. We also observe that the ST call blocking probability is higher for the large group scenario than for the small group scenario.

Fig. 4 shows how the mean reconnection probability γ changes as a function of the PT call arrival rate λ and the ST call residence time $E[\tau]$ in the small and large secondary group secnarios. We observe that γ decreases as λ increases, and increases as $E[\tau]$ increases. A higher volume of PT calls results in a smaller chance that a queued ST call reconnects to the system, while a longer maximum waiting time leads to a higher probability of reconnection.

V. Conclusion

We presented an analytical performance model of a wireless network with opportunistic spectrum access (OSA). The proposed system model includes the primary users, which own the licensed spectrum, and a limited

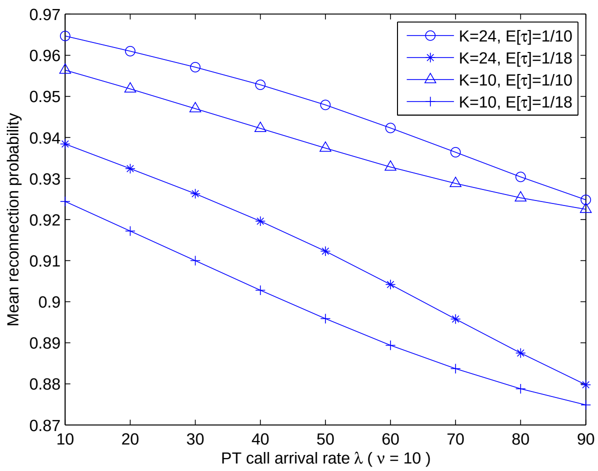

Fig. 4. The mean reconnection probability.

number of secondary users which opportunistically share the spectrum. The secondary users, equipped with cognitive radios, can sense the primary users and operate over a wide range of frequencies using appropriate communication mechanisms while causing negligible interference to the primary users. We evaluate the system performance using queuing analysis and obtain the steady state probability vector and some performance measures of interest. The proposed model and analysis method can be used to evaluate the performance of future opportunistic spectrum access systems.

In ongoing work, we are extending this study in a couple of directions. First, the analysis can be extended to accommodate variable bandwidth requirements of the primary users and secondary users. Future wireless networks will support multimedia applications with different resource requirements. Secondly, the analysis developed in this paper assumes that the secondary users are able to perfectly detect the presence or absence of a primary user on a channel. To make our model more applicable to real-world scenarios, the characteristics of an imperfect signal detection mechanism should be taken into account. Additional factors that would be included the analysis are detection errors, the interference impact to the primary users, etc.