AN EXACT SOLUTION FOR OUTAGE PROBABILITY IN CELLULAR NETWORKS

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Abstract— **We apply a discrete-time method to derive an exact solution for the outage probability in a cellular network as the mobile moves along a given trajectory, employing a hysteresis-based handoff algorithm. The exact solution addresses the relationships among the three key handoff parameters: hysteresis level, fade margin, and outage probability. Our numerical results, obtained from analysis and simulation, show that the proposed solution method is very accurate in both light and heavy shadowing environments, while previous approximate methods are accurate only in some specific environments.**[1]

I. Introduction

Future generation wireless networks are being designed to provide higher bandwidth and more seamless connectivity to mobile users. Handoffs occur whenever a mobile station (MS) moves from the coverage area of one base station (BS) to another. The handoff behavior of mobile stations in a cellular network has an important impact on the overall system performance. When a mobile moves in the vicinity of the cell boundaries, frequent handoffs between neighboring base stations will be executed. To avoid this undesirable ping-pong effect, hysteresis is typically introduced into the handoff decision process so that a handoff is made only when the received signal strength from one base station is higher than that from others by a given hysteresis level.

An important class of handoff algorithms operates on the basis of the *relative* processed signal strength between two candidate base stations [1]–[3]. In [1], an approximate model is developed for evaluating the performance of handoff algorithms based on relative signal strength measurements. This model was extended in [2] by taking into account the absolute signal strength from the current BS to reduce unnecessary handoffs. In [3], a discrete-time approach is introduced to analyze the handoff performance based on processed relative signal strength measurements. Exact analytical expressions are derived for cell assignment probability and handoff probability.

In a mobile wireless environment, the received signal at the mobile suffers impairment due to the path loss and the shadow fading. A fade margin is usually added to the transmit power to compensate for the impairment of the received signal so that the quality of the received signal at the MS is above an acceptable threshold for all but a certain fraction of time, called the outage probability. In [4], the effect of handoff techniques on cell coverage and reverse link capacity is studied for a CDMA system. A simple fade margin analysis is presented for both a soft handoff and a hard handoff system, and outage probability formulas are derived. However, the analysis of hard handoff systems focuses on single-cell coverage, i.e., the hysteresis level is infinity, which eliminates the important effect of diversity in hard handoff systems. Consequently, the fade margins required for such systems are over-estimated in [4].

In [5], an analysis of fade margins is presented for both soft handoff and hard handoff systems. Their analysis is based on a model for hard handoffs with hysteresis and connection delay. The approximation for fade margin obtained in [5] is more accurate than that of [4], although some approximations are used in their derivation of outage probability. In [6], a simulation study of fade margins is presented for two-cell hard handoff systems. The approach of [6] still leads to an over-estimation of the fade margin required for hard handoff, although their approximation is much more accurate than that of [4]. Sung [7] presents an alternative analysis of the effect of handoff techniques on fade margins. An upper bound on the fade margin for hard handoff systems is derived and an approximate formula for the outage probability is obtained empirically. The approximation formula is accurate in certain scenarios, for example, in light shadowing environments. However, it is not suitable in environments characterized by heavy shadowing.

In this paper, we apply a discrete-time method to derive an *exact* solution for the outage probability of a mobile station in a cellular network. In contrast to the aforementioned works on outage probability analysis, our approach captures the past history of the mobile connection state as the mobile moves along a given trajectory. The exact solution addresses the relationships among three key handoff parameters: hysteresis level, fade margin, and outage probability. Our numerical results, obtained from analysis and simulation, show

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that the proposed method leads to an accurate solution for outage probability irrespective of the propagation environment characteristics. Previous methods for computing the outage probability are only applicable in some specific environments, due to the use of approximations in their derivations.

The remainder of the paper is organized as follows. Section II describes the basic system model addressing radio signal propagation. Section III presents the hysteresis analysis for a class of handoff algorithm based on *relative* processed signal strength. Section IV presents an analysis of fade margin based on the above system model. Section V develops an analysis of outage probability along a given trajectory, which is evaluated in Section VI. Section VII presents numerical results obtained from the proposed method and provides comparisons with results obtained from simulation and earlier analytical approximations. Finally, the paper is concluded in Section VIII.

II. System Model

The cellular network consists of a set of base stations deployed in a coverage area. The MS makes handoff decisions based on measurements of the pilot signal strength received from the two nearest base stations, say BS_i and BS_j. Our analysis of handoff performance is based on a discrete-time model for the signal strength measurements. In the discrete-time model, the mobile unit samples the pilot signal strengths at time instants $t_k = kt_s$, where t_s is the sampling interval. The distance between successive sampling positions of the mobile is denoted by $d_s = vt_s$. The discrete-time model captures the handoff behavior of an MS more accurately than its continuous-time counterpart since all signal strength measurements are sampled, in practice.

In discrete-time, the measured pilot signal strength $S_i[k]$ from base station BS_i at time k can be expressed in dB as (cf. [3], [8])[2]:

$$S_i[k] = m_i[k] + W_i[k] + Z_i[k], \tag{1}$$

where $m_i[k]$, $W_i[k]$, and $Z_i[k]$ represent the path loss, shadowing and fast fading components, respectively. The path loss component $m_i[k]$ is given by

$$m_i[k] = S_{\text{tr},i} - 10\mu \log d_i[k], \tag{2}$$

where $S_{\text{tr},i}$ is the transmitted signal strength from BS_i in dB, μ is the path loss exponent with a typical value of 4 in a shadowed urban environment. The process $\{W_i[k]\}$ represents independent zero-mean, stationary Gaussian process characterized by an autocorrelation function given by (cf. [8], [10])

$$R_{W_i}(m) = \sigma^2_{W_i} \exp\left(\frac{-|m|d_s}{d_0}\right). \tag{3}$$

To eliminate the fast fading $Z_i[k]$, the received signal strength $S_i[k]$ is processed by an exponential smoothing window given by

$$f_{\text{av}}[k] = \frac{1}{d_{\text{av}}} \exp\left(\frac{-kd_s}{d_{\text{av}}}\right) = \frac{b^k}{d_{\text{av}}}, \qquad k \geq 0 \tag{4}$$

where $b = \exp(-d_s/d_{\text{av}})$. Hence, the processed pilot signal strength from base station BS_i is obtained as follows:

$$X_i[k] = f_{\text{av}}[k] * S_i[k], \qquad k \geq 0, \tag{5}$$

where $*$ denotes discrete-time convolution. It can be shown that $\{X_i[k]\}$ is a second-order autoregressive (AR) process [3], a fact which we exploit to compute outage probability.

III. Hysteresis Analysis

As mentioned earlier, hysteresis is typically introduced into the handoff decision process to avoid the undesirable ping-pong effect. A number of hysteresis-based handoff decision methods using received signal measurements have been investigated in the literature [1]–[3], [11]. The *relative* processed signal strength for handoff decision between base stations BS_i and BS_j is given by

$$X[k] \triangleq X_i[k] - X_j[k]. \tag{6}$$

A handoff of the mobile user from BS_i to BS_j occurs when the relative processed signal strength $X[k]$ falls below the value $-h_j$. Conversely, a handoff from BS_j to BS_i occurs when $X[k]$ exceeds the value h_i. The values h_i and h_j are known as the hysteresis levels associated with BS_i and BS_j, respectively.

In [3], handoff behavior is characterized in terms of the regions to which the relative signal strength is assigned at each value of the discrete-time parameter k. We define three *assignment regions*, which are intervals of the real-line:

$$I \triangleq [h_i, \infty], J \triangleq (-\infty, -h_j], H \triangleq (-h_j, h_i).$$

The assignment regions at system initialization time are defined as

$$I_0 \triangleq (0, \infty), J_0 \triangleq (-\infty, 0).$$

For conventional handoff, the mobile will be assigned to base station k if $X[k] \in I$ or if $X[l] \in I$ for some $l < k$ and $X[j] \in H$ for $l < j \leq k$. Let $X[k \backslash r]$ denote

[2]Because W_i has a density function symmetric about 0, it is also correct to write $-W_i[k]$ in equation (1) (cf. [9]).

the sequence consisting of the r most recent values of $\{X[k]\}$ up to and including time k, where $1 \leq r \leq k+1$:

$$X[k\backslash r] \triangleq (X[k–r+1], X[k–r+2], \cdots X[k]). \quad (7)$$

Equivalently, $X[k\backslash r]$ may be mapped to a string of length k on the alphabet

$$\mathcal{A} \triangleq \{I_0, I, J_0, J, H\}.$$

We shall use the notation H^r to denote a string consisting of the symbol H repeated r times. When $k = 0$, $X[k]$ falls in either region I_0 or region J_0. At any time $k \geq 1$, $X[k]$ falls in exactly one of the three assignment regions I, J, and H. Cell assignment of an MS along a given trajectory can be characterized as follows: for $k \geq 1$, if the MS is assigned to BS_i at time $k-1$, a handoff to cell j occurs at time k if and only if $X[k] \in J$. Conversely, if the MS is assigned to BS_j at time $k-1$, a handoff to cell i occurs at time k if and only if $X[k] \in I$.

Let $E_i[k]$ denote the event that the mobile is assigned to base station BS_i at time k. We refer to $E_i[k]$ as the cell i assignment event at time k. We can express the assignment event, $E_i[k]$, as a disjoint union of more elementary events[3] [3]:

$$E_i[k]=\{X[k\backslash k+1]\in I_0 H^k\} \sqcup \bigsqcup_{r=1}^{k} \{X[k\backslash r]\in I H^{r-1}\}. \quad (8)$$

Similarly, we can express the cell j assignment event, denoted by $E_j[k]$, as

$$E_j[k]=\{X[k\backslash k+1]\in J_0 H^k\} \sqcup \bigsqcup_{r=1}^{k} \{X[k\backslash r]\in J H^{r-1}\}. \quad (9)$$

IV. Fade Margin Analysis

In a mobile wireless environment, the received signal at the mobile suffers impairment due to the path loss and the shadow fading. In order for an acceptable quality of communications, the received signal strength must exceed a certain threshold, m_{RSS} (minimum received signal strength), which is a quality-of-service (QoS) parameter defined by the network provider. When there is no shadow fading, the minimum required transmitted signal strength is the propagation path loss plus m_{RSS}, where m_{RSS} is the received signal strength in this case. Thus, the transmitted signal strength as a function of the distance d from the base station is given by

$$S_{\text{tr}}(d) = m_{\text{RSS}} + 10\mu \log d. \quad (10)$$

[3]When sets A and B are disjoint we denote their union by $A \bigsqcup B$ to emphasize this property.

When there is shadow fading, however, the received signal strength is given as follows:[4]

$$S = S_{\text{tr}}(d) - 10\mu \log d - W, \quad (11)$$

where S is the received signal strength and W is the shadow fading component. In the presence of shadow fading, a fade margin γ should be added to the transmitted signal so that the received signal strength is guaranteed to be greater than m_{RSS} for all but a certain fraction of time, called the *outage probability*. That is,

$$S_{\text{tr}}(d) = \gamma + m_{\text{RSS}} + 10\mu \log d. \quad (12)$$

Comparing equations (11) and (12), we have the following relationship

$$S - m_{\text{RSS}} = \gamma - W. \quad (13)$$

Based on the above equations, we make the following important remarks:

- Without shadow fading, the minimum transmitted signal strength required to provide the desired QoS depends only on the distance d. As a result, the MS remains connected to the nearer base station. In a shadow fading environment, with the fade margin γ introduced, it is possible for the MS to be connected to the farther base station via a handoff event.
- Maintaining the fade margin greater than the shadow fading is equivalent to ensuring that the received signal strength is greater than m_{RSS}. Conversely, if the fade margin is not large enough to overcome the shadow fading, then the received signal will not satisfy the desired QoS requirement.

V. Outage Probability Analysis

Due to shadow fading, the received signal strength fluctuates so that even if the average received signal strength is above m_{RSS}, it is possible for the received signal strength to fall below m_{RSS} for some time periods. The fraction of time that the received signal strength is below m_{RSS} is the outage probability. The outage probability is an important QoS parameter that system designers attempt to keep below a certain threshold. An outage event can occur while the MS is assigned either to BS_i or BS_j. Let $Q_i[k]$ and $Q_j[k]$ denote the outage events at time k when the MS is assigned to cell i and cell j, respectively. A cell i outage event at time k occurs under the following conditions:

- at time $k-1$, the MS is assigned to BS_i; and
- at time k,

[4]Here we do not consider the effect of fast fading since fade margin is mainly introduced to overcome shadow fading. Moreover, the fast fading effect can be easily averaged out.

- the received signal strengths from both BS_i and BS_j are less than m_{RSS}, i.e., $\max\{S_i[k], S_j[k]\} < m_{\text{RSS}}$ (or, equivalently, $\min\{W_i[k], W_j[k]\} > \gamma$); or
- the received signal strength from BS_i is less than m_{RSS} and that from BS_j is greater than m_{RSS} ($S_i[k] < m_{\text{RSS}}, S_j[k] > m_{\text{RSS}}$), but the handoff condition is not satisfied, i.e., $S_j[k] - S_i[k] < h$ (or $W_i[k] - W_j[k] < h$).

Therefore, we can write

$$\begin{aligned} Q_i[j] &= E_i[k-1] \cap [\{\min(W_i[k], W_j[k]) > \gamma\} \sqcup \\ &(\{W_i[k]>\gamma\} \cap \{W_j[k]<\gamma\} \cap \{W_i[k]-W_j[k]<h\})] \\ &= E_i[k-1] \cap \{W_i[k] > \gamma\} \cap [\{W_j[k] > \gamma\} \sqcup \\ &(\{W_j[k]<\gamma\} \cap \{W_i[k]-W_j[k]<h\})] \\ &= E_i[k] \cap \{W_i[k] > \gamma\}. \end{aligned} \tag{14}$$

In the second equation of (14), the first and third terms ensure that at time k, the MS is not able to make a handoff to BS_j; it can only stay at BS_i (resulting in the event $E_i[k]$) until the outage event happens. Similarly, $Q_j[k]$ can be expressed by following the same procedure with j replacing i in the subscripts:

$$Q_j[k] = E_j[k] \cap \{W_j[k] > \gamma\}. \tag{15}$$

Let $o_i[k]$ and $o_j[k]$ denote the outage probabilities in cells i and j at time k, and define $L_i[k] \triangleq \{W_i[k] > \gamma\}$ and $L_j[k] \triangleq \{W_j[k] > \gamma\}$, respectively. Then, we have

$$o_i[k] \triangleq \Pr(Q_i[k]) = \Pr(E_i[k] \cap L_i[k]), \tag{16}$$

$$o_j[k] \triangleq \Pr(Q_j[k]) = \Pr(E_j[k] \cap L_j[k]). \tag{17}$$

Substituting (8) and (9) into the above equations, and noting that the events on the right-hand sides of (8) and (9) are mutually exclusive, we obtain

$$\begin{aligned} o_i[k] &= \Pr(\{X[k\backslash k{+}1] \in I_0 H^k\} \cap L_i[k]) \\ &\quad + \sum_{r=1}^{k} \Pr(\{X[k\backslash r] \in I H^{r-1}\} \cap L_i[k]), \end{aligned} \tag{18}$$

$$\begin{aligned} o_j[k] &= \Pr(\{X[k\backslash k{+}1] \in J_0 H^k\} \cap L_j[k]) \\ &\quad + \sum_{r=1}^{k} \Pr(\{X[k\backslash r] \in J H^{r-1}\} \cap L_j[k]). \end{aligned} \tag{19}$$

For convenience, we introduce the notation

$$o_k(\mathbf{S}, L_i) = \Pr\{X[k\backslash|\mathbf{S}|] \in \mathbf{S} \cap L_i[k]\},$$

where $\mathbf{S}$ is a string of symbols on the assignment alphabet $\mathcal{A}$ with length denoted by $|\mathbf{S}|$. Then, the cell i and cell j outage probabilities can be expressed as

$$o_i[k] = o_k(I_0 H^k, L_i) + \sum_{r=1}^{k} o_k(I H^{r-1}, L_i), \tag{20}$$

$$o_j[k] = o_k(J_0 H^k, L_j) + \sum_{r=1}^{k} o_k(J H^{r-1}, L_j), \tag{21}$$

and the overall outage probability $o[k]$ is given by

$$o[k] = o_i[k] + o_j[k]. \tag{22}$$

VI. Outage Probability Evaluation

In [3], a recursive procedure was developed to compute the assignment and handoff probabilities for hysteresis-based hard handoff algorithms. The recursive procedure is specified in terms of a sequence of bivariate functions defined below:

$$\begin{aligned} & g_1(x_0, x_1) \triangleq f_1(x_0, x_1), \\ & g_2(x_1, x_2) \triangleq \int_{I_0} g_1(x_0, x_1) f_2(x_2|x_0, x_1) dx_0, \\ & g_k(x_{k-1}, x_k) \triangleq \\ & \int_I f_{k-1}(x_{k-2}, x_{k-1}) f_k(x_k|x_{k-2}, x_{k-1}) dx_{k-2} + \\ & \int_H g_{k-1}(x_{k-2}, x_{k-1}) f_k(x_k|x_{k-2}, x_{k-1}) dx_{k-2}, \end{aligned}$$

for $k \geq 3$. Here, $f_k(x_{k-1}, x_k)$ denotes the joint density of $\{X[k-1], X[k]\}$ and $f_k(x_k|x_{k-2}, x_{k-1})$ is the conditional density of $X[k]$ given $\{X[k-1] = x_{k-1}\}$ and $\{X[k-2] = x_{k-2}\}$. Both densities are Gaussian and derived in the appendix of [3].

The probability $o_i[k]$ can be expressed in terms of the functions g_k as follows:[5]

$$\begin{aligned} o_i[k] &= o_k(I, L_i) + \iiint_{D_i I H} f_k(x_{k-1}, x_k) \\ & f_{i,k}(w_i|x_{k-1}, x_k) dx_{[k\backslash 2]} dw_i + \iiint_{D_i H^2} g_k(x_{k-1}, x_k) \\ & f_{i,k}(w_i|x_{k-1}, x_k) dx_{[k\backslash 2]} dw_i, \quad k \geq 2, \end{aligned} \tag{23}$$

where $D_i = (\gamma, \infty)$ and the initial condition is given by

$$o_i[1] = o_1(I_0 H, L_i) + o_1(I, L_i).$$

The probability $o_k(I, L_i)$ is given by

$$\begin{aligned} o_k(I, L_i) = & \iiint_{D_i \mathcal{R} I} f_k(x_{k-1}, x_k) \\ & f_{i,k}(w_i|x_{k-1}, x_k) dx_{[k\backslash 2]} dw_i, \quad k \geq 1, \end{aligned} \tag{24}$$

[5]See Appendix I for the proof of equivalence between (23) and (20) and an explanation of the compact notation used for integrals.

where $\mathcal{R} \triangleq (-\infty, \infty)$, and $f_{i,k}(w_i|x_{k-1}, x_k)$ is the conditional density of $W_i[k]$ given $X[k]$ and $X[k-1]$. An expression for this density function is given in Appendix II. The probability $o_j[k]$, can be expressed similarly as in the above equation by replacing i, I, and D_i with j, J, and D_j, respectively.

VII. Numerical Results and Discussions

In this section, we present numerical results to validate the accuracy of our analytical technique for computing the outage probability. The main system parameters are set as follows: the distance between the current BS and the candidate BS is set to be $D = 2000$ m, the path loss exponent $\mu = 4$, $\sigma_{W_i} = \sigma_{W_j} = \sigma_W = 8$ dB, $d_o = 20$ m, $d_s = 1$ m, $m_{\text{RSS}} = -96$ dB. The averaging parameter d_{av} is set to 10 m. The hysteresis levels for the base stations are set to the same value h, i.e., $h_i = h_j = h$. We first evaluate the performance of outage probability with respect to hysteresis and fade margin as the MS moves along the straight line between the two base stations. Then we compare our exact solution approach with approximation solution approaches developed by Chopra [6] and Sung [7], under light and heavy shadowing environments.

In Fig. 1, we observe that the analytical outage probability curves increase as the hysteresis level h increases. The analytical results are validated by simulation curves, which show the 95% confidence intervals obtained from running 10,000 trials. The results are intuitive, because the larger the hysteresis level, the larger the handoff delay and the more likely an outage event occurs. We also observe that along the straight line trajectory between the two base stations BS_i and BS_j, the outage probability first increases until around the midpoint between the two base stations and then decreases beyond this midpoint. This is because the outage event happens more frequently around the midpoint between the two base stations, where the received signal strengths are smaller.

In Fig. 2, we observe that the analytical outage probability curves increase as the fade margin γ decreases, which is also verified by the simulation results. The reason is that the smaller the fade margin, the smaller the received signal at the MS and the more likely outage events occur. We also observe the same trend of the outage probability curves as that in Fig. 1 when the MS moves along a given trajectory between BS_i and BS_j.

Figs. 3 and 4 compare our exact approach for computing outage probability with the earlier approximations of Chopra [6] and Sung [7] under the assumption of no site-to-site correlation. For a fair comparison, we diminish the effect of the exponential smoothing window by choosing an appropriate value of d_{av}, since no smoothing window was considered in [7]. We consider a *light* and a *heavy* shadowing environment by setting the standard deviation of the shadowing noise to 8 dB and 12 dB, respectively. In both figures, the y-axis represents the value of the outage probability averaged over the interval $[900, 1100]$, in the vicinity of the midpoint along the straight line trajectory from BS_i to BS_j. The x-axis represents the value of the fade margin in dB. The exact solutions are validated by simulation curves showing the 95% confidence intervals obtained from 10,000 trials.

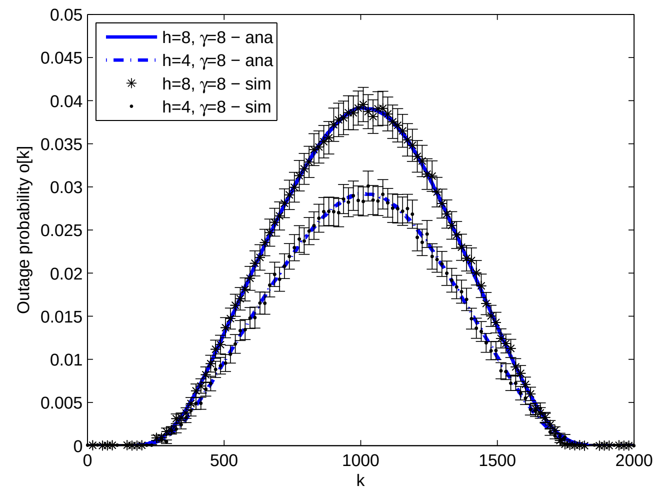

Fig. 1. The effect of hysteresis on outage probability.

From Fig. 3, we observe that in a light shadowing environment, Sung's approximation [7] is very close to our exact solution, whereas Chopra's approximation [6] is notably less accurate. By contrast, Fig. 4 shows that in a heavy shadowing environment, Chopra's approximation is closer to our exact solution than Sung's approximation. The inaccuracies found in the approximations of Sung and Chopra are due to the fact that they do not take into account the past history of the MS connection state. In contrast, the proposed method takes into account the MS connection state history and is accurate irrespective of the shadowing characteristics. Under the considered system configuration, Sung's approximation is accurate in light shadowing environments while Chopra's approximation is accurate in heavy shadowing environments.

VIII. Conclusion

We have presented an exact discrete-time method for computing the outage probability for hysteresis-based handoff on the basis of the past history of the mobile connection state as the mobile moves along an arbitrary trajectory. Our analysis addresses the relationships among the three key parameters: hysteresis

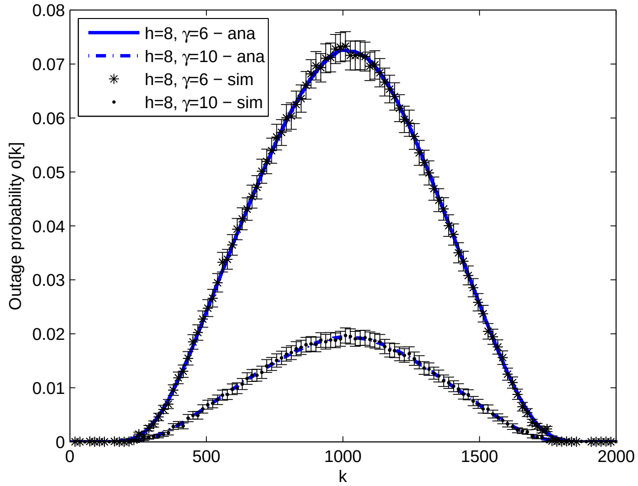

Fig. 2. The effect of fade margin on outage probability.

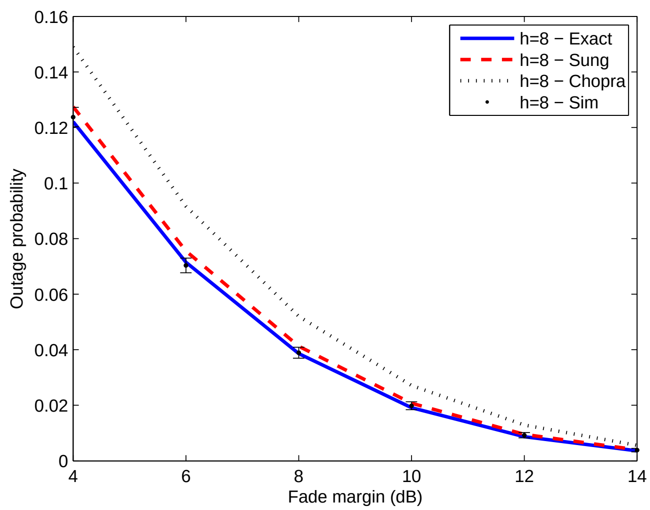

Fig. 3. Outage probability vs. fade margin in a light shadowing environment ($\sigma_W = 8$ dB).

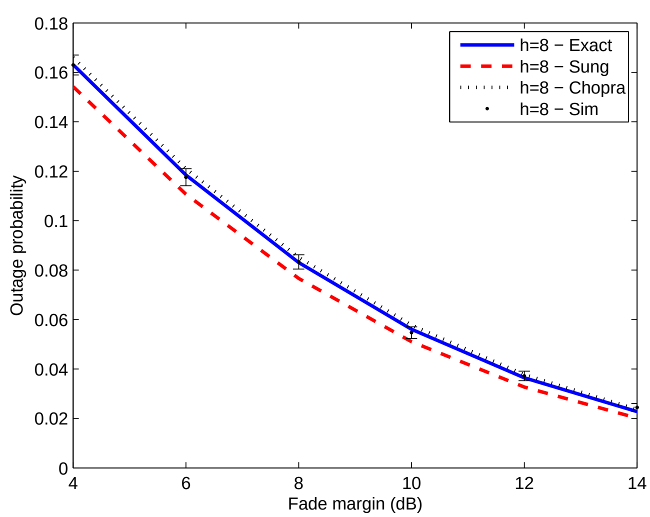

Fig. 4. Outage probability vs. fade margin in a heavy shadowing environment ($\sigma_W = 12$ dB).

level, fade margin, and outage probability. Our numerical results, obtained using analysis and simulation, show that the proposed method is accurate irrespective of the shadowing characteristics, whereas previous analytical approaches are applicable only under certain conditions of shadowing.

APPENDIX I
EQUIVALENCE BETWEEN (23) AND (20):

We first introduce the following compact notation for a multidimensional integral over the intervals $A_{k-n+1}, \ldots, A_k$, of the real line ($k \geq n-1$) [3]:

$$\int_{A_{k-n+1}\ldots A_k} g(x_{[k\backslash n]})dx_{[k\backslash n]} \triangleq$$
$$\int_{A_{k-n+1}} \cdots \int_{A_k} g(x_{k-n+1}, \ldots, x_k)dx_k \ldots dx_{k-n+1}.$$

The equivalence between (23) and (20) is shown as follows:

$$\begin{aligned}
&o_i[k] = o_k(I, L_i) + o_k(IH, L_i) \\
&+ \int_{D_i IH^2} f_{k-1}(x_{[k-1\backslash 2]}) f_k(x_{[k-1\backslash 2]}) \\
&\cdot f_{i,k}(w_i|x_{[k\backslash 2]})dx_{[k\backslash 3]}dw_i \\
&+ \int_{D_i H^3} g_{k-1}(x_{[k-1\backslash 2]}) f_k(x_k|x_{[k-1\backslash 2]}) \\
&\cdot f_{i,k}(w_i|x_{[k\backslash 2]})dx_{[k\backslash 3]}dw_i \\
&= o_k(I, L_i) + o_k(IH, L_i) + o_k(IH^2, L_i) \\
&+ \int_{D_i IH^3} f_{k-2}(x_{[k-2\backslash 2]}) f_{k-1}(x_{k-1}|x_{[k-2\backslash 2]}) \\
&\cdot f_k(x_k|x_{[k-2\backslash 2]}) f_{i,k}(w_i|x_{[k\backslash 2]})dx_{[k\backslash 4]}dw_i \\
&+ \int_{D_i H^4} g_{k-2}(x_{[k-2\backslash 2]}) f_{k-1}(x_{k-1}|x_{[k-2\backslash 2]}) \\
&\cdot f_k(x_k|x_{[k-1\backslash 2]}) f_{i,k}(w_i|x_{k-1}, x_k)dx_{[k\backslash 4]}dw_i \\
&= \cdots \\
&= o_k(I, L_i) + o_k(IH, L_i) + o_k(IH^2, L_i) + \cdots \\
&+ o_k(IH^{k-1}, L_i) + \int_{D_i H^k} g_2(x_{[2\backslash 2]}) f_3(x_3|x_{[2\backslash 2]}) \\
&\cdots f_k(x_k|x_{[k-1\backslash 2]}) f_{i,k}(w_i|x_{[k\backslash 2]})dx_{[k\backslash k]}dw_i \\
&= \sum_{r=1}^{k} o_k(IH^{r-1}, L_i) + o_k(I_0 H^k, L_i).
\end{aligned}$$

Note that the above derivation uses the second-order Markov property of the process $\{X[k]\}$, mentioned in Section II.

Appendix II
Derivation of $f_{i,k}(w_i|x_{k-1}, x_k)$

Let $S[k] \triangleq S_i[k] - S_j[k]$. From (5), we obtain

$$S[k] = d_{\text{av}}(X[k] - bX[k-1]).$$

The conditional cumulative distribution function can be expressed as

$$\begin{aligned}
&F_{i,k}(w|x_{k-1}, x_k) \\
&= 1 - \Pr\{W_{i,k} > w|X_{[k\backslash 2]} = x_{[k\backslash 2]}\} \\
&= 1 - \Pr\{S_i[k] < \gamma + m_{\text{RSS}} - w|X_{[k\backslash 2]} = x_{[k\backslash 2]}\} \\
&= 1 - \Pr\{S_j[k] < \gamma + m_{\text{RSS}} - w - d_{\text{av}}(x_k - bx_{k-1})\} \\
&= 1 - F_{S_j[k]}(\gamma + m_{\text{RSS}} - w + d_{\text{av}}(bx_{k-1} - x_k)).
\end{aligned}$$

Taking the first derivative on both sides of the last equation, we obtain

$$\begin{aligned}
&f_{i,k}(w|x_{[k\backslash 2]}) \\
&= f_{S_j[k]}(\gamma + m_{\text{RSS}} - w + d_{\text{av}}(bx_{k-1} - x_k)) \\
&\sim \mathcal{N}(\gamma + m_{\text{RSS}} + d_{\text{av}}(bx_{k-1} - x_k) - m_{S_j}, \sigma^2_{S_j}),
\end{aligned}$$

where $\mathcal{N}(\mu, \sigma^2)$ denotes the Gaussian density with mean μ and variance σ^2, and m_{S_j} and σ_{S_j} are, respectively, the mean and standard deviation of $S_j[k]$.

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