Game-Theoretic Framework for Cooperative Relaying in Cognitive Radio Networks

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***Abstract*—We propose a novel Stackelberg game-theoretic framework to jointly manage spectrum resources and coordinate secondary users in a cognitive radio network with decode-and-forward cooperative relaying capability to extend coverage. The primary users (PUs) and secondary users (SUs) are mapped into leader-follower pairs in which the SUs purchase spectrum resources from their corresponding PU leaders. An optimal SU transmit strategy incorporating cooperative relaying is derived, and a hybrid scheduling algorithm incorporating both direct transmission and relay transmission is proposed. Experimental results show that the proposed Stackelberg game framework can achieve significantly better system performance with cooperative relaying compared to an SU direct transmission scheme.[1]**

***Index Terms*—Cognitive radio, dynamic spectrum access, cooperative relaying, decode-and-forward, Stackelberg game.**

I. Introduction

With the rapid growth of wireless communications, spectrum scarcity has become an important issue. Cognitive radio is a promising technology that aims to improve spectrum utilization by allowing unlicensed secondary users (SUs) to access and share licensed spectrum that is not being used by the primary users (PUs) [1]–[3]. Cognitive radio networks (CRNs) require suitable power control and resource allocation schemes to avoid harmful interference to the PUs and promote system capacity. With proper interference coordination, the SUs can efficiently access the available spectrum and achieve higher system capacity without compromising the PUs.

The SUs form a CRN either through a base station [2], [3] or via direct communications [1], whereby the SUs establish direct links or adopt other SUs as relays to transmit information. Communications using direct links among SUs could be an effective alternative to an infrastructured network, where the SUs form an underlay ad-hoc network with multiple link requirements. Meanwhile, as the SUs might be far apart from each other, relay SUs are often necessary to support transmission. In addition, spectrum resource allocation becomes increasingly complex as the communication demand grows. Hence, spectrum resource allocation in an SU underlay network emerges as a research challenge.

Game theory has been proposed as an effective mathematical tool to model the interactions among network devices and predict their future actions [4]–[6]. In CRNs, the SUs interact with the PUs to compete for access spectrum resources. By assuming that the PUs and SUs are independent rational players, their transmission strategies can be derived. Among a variety of game-theoretic models, Stackelberg games have been intensively studied [7], [8]. Yet existing Stackelberg frameworks model only the interactions between PUs and SUs in a direct transmission scheme, whereas the potential benefit of cooperative relaying has not been considered in this context. The Stackelberg game has also been applied to model the interactions between different hops in cooperative relaying in [9], [10], but spectrum allocation is not considered in their model. To address the above concerns, a novel game framework is required to characterize the interactions between SUs in cooperative relaying and the PUs.

In this paper, we assume that certain transmission demands exist among a number of SU pairs, and we model such interactions using an extended Stackelberg game framework. Our main contributions are as follows:

- We propose a novel framework for downlink transmission with SU cooperative relaying under a decode-and-forward (DF) scheme, in which the interactions between the devices in cooperative relaying (i.e., the SUs) and the potential spectrum providers (i.e., the PUs) are modeled by a Stackelberg game. Theoretical analysis of the Nash equilibrium of the Stackelberg game is carried out to derive the optimal transmission strategies of the devices.
- Based on the game outcomes, we design a hybrid priority-based scheduling scheme to jointly select appropriate relay nodes for SU pairs and allocate spectrum resource for both direct transmission (DT) and DF relaying.
- We analyze the performance of the proposed scheduling scheme by running extensive simulations. Our numerical results show that significant performance improvement can be achieved by employing DF relaying among the SUs compared to using DT only.

The remainder of the paper is organized as follows. In Section II, the system model adopted in this paper is described. Section III presents the analysis for DF cooperative relaying strategy and scheduling approach under the Stackelberg game framework. In Section IV, simulation results and analysis are presented. Concluding remarks are given in Section V.

II. System Model

A. Cognitive Radio Network Model

We shall assume a simplified cognitive radio network (CRN) model consisting of one base station (BS), multiple primary

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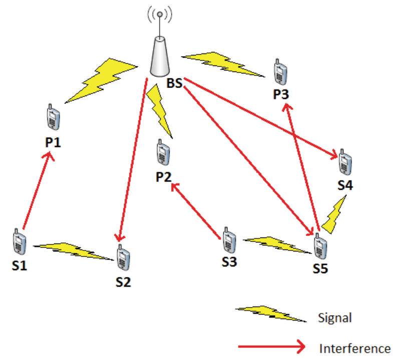

Fig. 1. CRN model with downlink channel sharing.

users (PUs) and multiple secondary users (SUs), which are deployed over a network coverage area. The BS, PUs and SUs are each equipped with an omni-directional antenna. Each PU is assigned a fixed licensed channel for communication by the BS, and the SUs are allowed to access these channels as long as they are detected idle. The SUs comprise SU transmitter-receiver pairs with certain communication demands and independent SUs. The independent SUs may act as cooperative relays even when a transmitter and receiver pair are too far apart to set up a direct link.

We focus on downlink channel access and consider the effect of co-channel interference between PU and SU nodes. A scenario illustrating the proposed spectrum sharing scheme in a cognitive network is presented in Fig. 1. For simplicity, we assume that at most one relay node can be adopted by each SU pair. As depicted in Fig. 1, the SU pair (S_1, S_2) uses DT, which shares the channel that has been licensed to PU P_1, and S_2 suffers interference from the BS. Meanwhile, PU P_1 receives interference from S_1. As a contrast, the SU pair (S_3, S_4) uses DF transmission using SU S_5 as the relay. We assume half-duplex transmission and that the two hops do not use the same channel and hence do not cause interference to each other. The PUs P_2 and P_3 are selected using spectrum sensing (cf. [11]) to provide the channel resources to the first and the second hop, respectively. Apparently, S_4 receives only the signal forwarded from S_5.

B. Decode-and-Forward Transmission

In DF strategy, the relay node fully decodes the signal received from the previous hop and then forwards the signal to the next hop [4]. Time division multiplexing (TDM) is assumed, such that either the SU transmitter or the relay node can transmit during each transmission time interval (TTI). In the forwarding process, the relay node selects a different channel and transmits the signal using an appropriate power. We assume that at most one relay node could be used in each SU end-to-end link. Since the relay node needs extra processing effort to decode the signal, the DF relaying scheme takes two TTIs to complete.

In our model, the BS uses equal transmit power to each PU. The transmit power of the BS, the SU and the relay node are denoted by p_b, p_s, p_r, respectively. In TDM transmission, since no interference occurs in DF relay transmission, the two hops could either access the same channel or different channels. In general, if the SU transmitter and the relay node access the channel provided by PU i and PU j respectively in this scenario, the signal-to-noise-plus-interference ratio (SINR) received by the PUs are given by:

$$\gamma_{p,i} = \frac{p_b g_{bp,i}}{p_s g_{sp} + N_0}, \quad \gamma_{p,j} = \frac{p_b g_{bp,j}}{p_r g_{rp} + N_0} \tag{1}$$

where $g_{bp,i}, g_{bp,j}, g_{sp}, g_{rp}$ are the channel coefficients from the BS to PUs i and j, from the SU transmitter to PU i, and from the relay node to PU j, respectively. The additive channel noise power is denoted by N_0. Similarly, the SINR received by the relay node and the SU transmit power are given by

$$\gamma_r = \frac{p_s g_{sr}}{p_b g_{br} + N_0}, \quad \gamma_s = \frac{p_r g_{rs}}{p_b g_{bs} + N_0}, \tag{2}$$

where $g_{sr}, g_{br}, g_{rs}, g_{bs}$ denote the channel coefficients from SU transmitter to relay, from BS to relay, from relay to SU receiver and from BS to SU receiver, respectively. For any devices i, j, the channel coefficient is given by $g_{ij} = |h_{ij}|/d_{ij}^2$, where d_{ij} is the distance between device i and j, and h_{ij} follows a complex Gaussian distribution (cf. [8]).

III. Stackelberg Game Model and Analysis

A. Stackelberg Game Model

In the system model, the SUs act as the underlay to the primary system, and we focus on power control and channel scheduling for the SUs pairs and relay nodes. In power control, an SU pair aims to find the optimal transmit strategy based on its interactions with the PUs. Then the SU pairs are mapped to the PUs so as to optimize overall network performance.

Stackelberg game is a powerful tool to describe the interactions between two rational players by modeling them as a leader-follower pair [8], in which both the leader and the follower seek to maximize their own utilities. Unlike DT, the *leader* consists of the two PUs, while the *follower* consists of the SU pair and the relay node. We assume Rayleigh fading channel model for the network, and that the relay node has the same transmit power range as the SU transmitter, i.e., $[p_{\min}, p_{\max}]$. From Shannon's capacity formula, the rates of the two hops are given by $r_1 = \log_2(1+\gamma_r)$, $r_2 = \log_2(1+\gamma_s)$, respectively. Similarly, the rates of the two PUs are given by $r_{p,i} = \log_2(1+\gamma_{p,i}), i = 1, 2$. Clearly, the hop with the smaller channel rate is the bottleneck. We further assume that the charging prices are the same for accessing both channels, which can be denoted by α_d. The leader gains revenue by selling its spectrum to the follower and its utility function can be written as

$$u_{l,r}(\alpha_d, p_s, p_r) = r_{p,1} + r_{p,2} + \alpha_d \beta p_s g_{sp} + \alpha_d \beta p_r g_{rp}, \tag{3}$$

where β is the ratio between the leader's revenue and the follower's payment. Clearly, the hop with the smaller channel rate determines the overall rate in DF transmission, and it takes two TTIs to transmit under the DF scheme. Therefore, the follower utility can be written as

$$u_{f,r}(\alpha_d, p_s, p_r) = \frac{1}{2}\min\{r_1, r_2\} - \alpha_d p_s g_{sp} - \alpha_d p_r g_{rp}. \quad (4)$$

The link capacity depends on the channel parameters as well as BS transmit power. Since the SU receiver will not receive interference from the first hop, either of the two hops could be the bottleneck in relay transmission, and we select the minimum channel rate among the two hops. The leader first decides the charging price α_d for the spectrum, and the follower then decides the transmit powers (p_s, p_r) based on the prices for the SU transmitter and relay node, respectively. Stackelberg game equilibrium is achieved when no unilateral deviation of α_d^* and (p_s^*, p_r^*) leads to higher leader utility or follower utility, i.e.,

$$u_{f,r}(\alpha_d^*, (p_s^*, p_r^*)) \geq u_{f,r}(\alpha_d^*, (p_s, p_r)) \quad (5)$$

$$u_{l,r}(\alpha_d^*, (p_s^*(\alpha_d^*), p_r^*(\alpha_d^*))) \geq u_{l,r}(\alpha_d^*, (p_s(\alpha_d^*), p_r(\alpha_d^*))) \quad (6)$$

From (4), one sees that calculating the link rate is a key issue. Define

$$C := \frac{\gamma_r}{\gamma_s} \text{ and } \rho := \frac{p_{\min}}{p_{\min} + p_{\max}}. \quad (7)$$

Since both p_s and p_r are between $p_{\min}$ and $p_{\max}$, we know that $\rho \leq p_s/p_r \leq \rho^{-1}$, and the bottleneck rate r^* of the two hops can be determined by comparing p_s/p_r and C. Depending on the channel parameters, there are four exhaustive and mutually exclusive cases:

- *Case 1*, $C \geq \rho^{-1}$: Clearly $r_1 \leq r_2$, so $r^* = r_1$.
- *Case 2*, $1 \leq C \leq \rho^{-1}$: Either r_1 or r_2 could be the bottleneck;
- *Case 3*, $\rho^{-1} \leq C \leq 1$: Either r_1 or r_2 could be the bottleneck;
- *Case 4*, $C \leq \rho$: In this case, $r_1 \geq r_2$, so $r^* = r_2$.

Since the (p_r, p_s) parameter space is symmetric, we need only analyze Cases 1 and 2. The analyses for Cases 3 and 4 can be derived in the same way. In the Stackelberg game, the leader knows that the follower will react to its behavior. Thus, it analyzes all the possible follower strategies and asserts the optimal price. The follower subsequently decides the optimal SU and relay transmit power based on the optimal price.

B. Analysis for Case 1

Similar to the analysis in [8], the optimal charging price, denoted by $\alpha_{d,1}^*$, is selected from the set $\{\phi_1, \phi_2, \phi_3\}$, where[2]:

$$\phi_1 = \frac{2B_1}{\beta(N_0 - D_1)} - \frac{B_1}{A_1},\ \phi_2 = \frac{B_1}{A_1} - \frac{2B_1}{\beta(N_0 + A_1 - D_1)},$$

$$\phi_3 = \frac{-B_1(A_1 + 2C_1) + \sqrt{A_1^2 B_1^2 - \frac{8A_1 B_1^2 C_1(A_1+C_1)}{\beta(C_1 - N_0 + D_1)}}}{2C_1(A_1 + C_1)} \quad (8)$$

and

$$A_1 = p_b g_{bp,1}, B_1 = \frac{1}{2\ln 2}, C_1 = \frac{-g_{sp}}{\gamma_r}, D_1 = p_{\min} g_{rp}. \quad (9)$$

[2]Details are omitted due to space constraints.

Based on the the optimal price, the optimal SU and relay transmit power are given by

$$p_{s,1}^* = \frac{1}{2\alpha_{d,1}^* g_{sp} \ln 2} - \frac{1}{\gamma_r},\ p_{r,1}^* = p_{\min}, \quad (10)$$

respectively. The leader and follower utilities in Case 1 are given by $u_{l,r,1} = u_{l,r}(\alpha_{d,1}^*, p_{s,1}^*, p_{r,1}^*)$ and $u_{f,r,1} = u_{f,r}(\alpha_{d,1}^*, p_{s,1}^*, p_{r,1}^*)$, respectively.

C. Analysis for Case 2

In Case 2, where $1 \leq C \leq \rho^{-1}$, either hop could be the bottleneck, depending on the channel coefficients and the threshold C. Hence, to derive the Stackelberg game equilibrium in Case 2, we need to find the optimal transmit power that maximizes the utility functions. It is easy to verify that in the search space, the straight line $p_s = Cp_r$ intersects $p_r = p_{\min}$ at $p_s = Cp_{\min}$. Therefore, the entire search space can be divided into two non-overlapping rectangular sub-areas, i.e., $p_s \in [p_{\min}, Cp_{\min}], p_r \in [p_{\min}, p_{\max}]$ and $p_s \in [Cp_{\min}, p_{\max}], p_r \in [p_{\min}, p_{\max}]$. To calculate the optimal transmit power and utility functions in Case 2, it is necessary to apply Stackelberg game analysis within both rectangular sub-areas, i.e.,

1) If $p_s \in [p_{\min}, Cp_{\min}]$, the first hop becomes the bottleneck.
2) If $p_s \in [Cp_{\min}, p_{\max}]$, either hop could be the bottleneck.

In each sub-area, an optimal price that maximizes leader utility uniquely exists and the SU transmit strategy can be uniquely decided. We compare Stackelberg game outcomes in both sub-areas and select the optimal transmit strategy (p_s, p_r) that maximizes the utility functions as the final outcome of Case 2. The analysis for the above scenarios is given as follows.

1) Scenario 1: In sub-area 1, the Stackelberg game model is the same as in Section III-B, except that the upper bound for p_s is $Cp_{\min}$ instead of $p_{\max}$. Therefore, the optimal SU transmit power is searched within $[p_{\min}, Cp_{\min}]$, which is denoted by $p_{s,2,1}^*$ in Scenario 1. The optimal relay transmit power $p_{r,2,1}^*$ in Scenario 1 takes the minimum value $p_{\min}$.

2) Scenario 2: When $p_s \geq Cp_{\min}$, either of the two hops could be the bottleneck. If the optimal transmit strategy satisfies $p_s \leq Cp_r$, the first hop becomes the bottleneck and the follower utility becomes

$$u_{f,r,2}(\alpha_d, p_s, p_r) = \frac{1}{2}r_1 - \alpha_d p_s g_{sp} - \alpha_d p_r g_{rp}. \quad (11)$$

Since $p_r \geq \frac{p_s}{C}$, we know that $-\alpha_d p_r g_{rp} \leq -\frac{\alpha_d p_s g_{sp}}{C}$, and we have $u_{f,r,2} \leq r_1/2 - \alpha_d p_s \gamma$, where $p_s \in [Cp_{\min}, p_{\max}]$ and $\gamma := g_{sp,1} + g_{rp,2}/C$. Clearly, the follower utility is maximized when the relay transmit strategy (p_s, p_r) is searched on the straight line $p_s = Cp_r$.

If $p_s \geq Cp_r$, the second hop is the bottleneck. Hence, the follower utility becomes

$$u_{f,r,2}(\alpha_d, p_s, p_r) = \frac{1}{2}r_2 - \alpha_d p_s g_{sp} - \alpha_d p_r g_{rp}. \quad (12)$$

Similarly, we know $-\alpha_d p_s g_{sp} \leq -\alpha_d p_r g_{rp} C$, and thus we have $u_{f,r,2} \leq r_2/2 - \alpha_d p_r C\gamma$. The relay transmit strategy

(p_s, p_r) is searched on the straight line $p_s = Cp_r$ as well. Therefore, the follower utility in Scenario 2 is maximized when $p_s = Cp_r$. We substitute this relation into (11) and set its first order derivative to 0, i.e., $\partial u_{f,r,2}/\partial p_s = 0$. The optimal SU transmit power in Scenario 2 is then derived as

$$p_{s,2,2} = \frac{1}{2\alpha_d \gamma \ln 2} - \frac{1}{\gamma_r} \tag{13}$$

and the corresponding relay transmit power is $p_{r,2,2} = p_{s,2,2}/C$. From the leader's perspective, the charging price α_d should be set neither too high nor too low to prevent insufficient outcome. Similar to the analysis in [8], α_d is set so that $Cp_{\min} \le p_{s,2,2} \le p_{\max}$ holds, and the lower and upper bound for α_d in Scenario 2 are denoted by $\alpha_{d\min,2,2}$ and $\alpha_{d\max,2,2}$, respectively. To derive the optimal α_d in Scenario 2, we substitute $p_{s,2,2}$ and $p_{r,2,2}$ into the leader utility (3), i.e.,

$$\begin{aligned} u_{l,r,2}(\alpha_d) &= \frac{\beta}{2\ln 2} - \frac{\alpha_d \beta \gamma}{\gamma_r} \\ &+ \log_2\left[1 + p_b g_{bp,1}\left(\frac{g_{sp}}{2\alpha_d \gamma \ln 2} + C_1\right)^{-1}\right] \\ &+ \log_2\left[1 + p_b g_{bp,2}\left(\frac{g_{rp}}{2\alpha_d C \gamma \ln 2} + C_2\right)^{-1}\right], \end{aligned} \tag{14}$$

where C_1 is as defined in (9) and we define

$$C_2 := -g_{rp,2}\frac{p_b g_{br,1} + N_0}{C g_{sr,1}} + N_0. \tag{15}$$

To maximize the leader utility (14), we take its first order derivative and set it to zero, i.e., $\partial u_{l,r,2}(\alpha_d)/\partial \alpha_d = 0$. It is easy to verify that $\partial u_{l,r,2}(\alpha_d)/\partial \alpha_d = 0$ yields a quartic polynomial equation. It is known that the analytical expression for the roots exist in such equation, and the real roots can be searched in $[\alpha_{d\min,2,2}, \alpha_{d\max,2,2}]$. Naturally, there are at most two local maxima in (14) and we denote them by $\hat{\alpha}^1_{d,2,2}, \hat{\alpha}^2_{d,2,2}$. Hence, the optimal price $\alpha^*_{d,2,2}$ in Scenario 2 is searched from the set $\left\{\hat{\alpha}^1_{d,2,2}, \hat{\alpha}^2_{d,2,2}, \alpha_{d\min,2,2}, \alpha_{d\max,2,2}\right\}$, the optimal SU transmit power in Scenario 2 becomes

$$p^*_{s,2,2} = \frac{1}{2\ln 2} \cdot \frac{1}{\alpha^*_{d,2,2}\gamma} - \frac{1}{\gamma_r}, \tag{16}$$

and the corresponding optimal relay node transmit power is $p^*_{r,2,2} = p^*_{s,2,2}/C$.

3) Optimal Strategy in Case 2: The optimal strategy in Case 2 can be derived by summarizing Scenarios 1 and 2. Since the leader attempts to profit as much as possible by sharing its licensed channels to the follower, we select the Stackelberg outcomes each of the two scenarios such that the leader utility in Case 2 is maximized, i.e.,

$$i^* = \underset{i \in \{1,2\}}{\arg\min} \quad u_{l,r}(\alpha^*_{d,2,i}, p^*_{s,2,i}, p^*_{r,2,i}) \tag{17}$$

and the optimal charging price, SU transmit power and relay node transmit power in Case 2 are denoted by $\alpha^*_{d,2}, p^*_{s,2}, p^*_{r,2}$, respectively. The leader and follower utilities in Case 2 are given by $u_{l,r,2} = u_{l,r}(\alpha^*_{d,2}, p^*_{s,2}, p^*_{r,2})$ and $u_{f,r,2} = u_{f,r}(\alpha^*_{d,2}, p^*_{s,2}, p^*_{r,2})$, respectively.

D. Optimal DF Transmission Strategy

Using the same approach, Stackelberg game outcomes can be derived for Cases 3 and 4, which are denoted by tuples $(\alpha^*_{d,3}, p^*_{s,3}, p^*_{r,3})$ and $(\alpha^*_{d,4}, p^*_{s,4}, p^*_{r,4})$, respectively. The follower and leader utilities in Cases 3 and 4 are denoted by $u_{f,r,3}, u_{l,r,3}$ and $u_{f,r,4}, u_{l,r,4}$, respectively. Hence, the optimal transmission strategy is selected from the tuples $(\alpha^*_{d,i}, p^*_{s,i}, p^*_{r,i})$ while the follower and leader utilities are selected from $\{u_{f,r,i}, u_{l,r,i}\}$, for $i \in \{1,2,3,4\}$.

E. Joint Scheduling and Allocation Algorithm

We propose a hybrid priority-based scheduling algorithm to jointly assign relay nodes and allocate channels to each SU pair regarding DF transmission. The SU pairs may either select DT or DF transmission, and the priorities are all based on follower utilities. The follower utility in DT is adopted from [8]. Since follower utilities for DT and DF transmission cannot be compared directly, we maintain two separate queues for DT and DF tranmission priorities denoted as queue 1 and queue 2, respectively. For the kth SU pair using the channel provided by ith PU, the priority in queue 1 is defined as $P^{\text{dt}}_{i,k} = u^{\text{dt}}_{i,k}$, where $u^{\text{dt}}_{i,k}$ is the optimal follower utility in DT. The corresponding link capacity is denoted by $C^{\text{dt}}_{i,k}$. Otherwise, if the kth SU uses the lth relay node to support DF transmission and access the channels provided by ith and jth PU, the priority in queue 2 is defined as $P^{\text{df}}_{i,j,k,l} = u^{\text{df}}_{i,j,k,l}$ where $u^{\text{df}}_{i,j,k,l}$ is the follower utility in DF transmission. The corresponding link capacity is denoted by $C^{\text{df}}_{i,j,k,l}$.

We initialize the two queues by calculating the priorities for all possible combinations of leader-follower pairs and sort them in descending order for both DT and DF transmission. During the scheduling process, we take the head of the two queues and compare their link capacities. The leader-follower pair with a larger link capacity is scheduled and all the devices scheduled are recorded. After this, we delete the head of both queues and all the leader-follower pairs that use any of the recorded devices in each queue. The scheduling process terminates when all the SU pairs have been scheduled. The algorithm is summarized in Algorithm 1.

Algorithm 1 Hybrid Scheduling Algorithm

```
Initialize P^dt_{i,k} and calculate C^dt_{i,k}, ∀i, k.
Initialize P^df_{i,j,k,l} and calculate C^df_{i,j,k,l}, ∀i, j, k, l.
repeat
    Select queue heads (i_1, k_1) and (i_2, j_2, k_2, l_2)
    if C^dt_{i_1,k_1} > C^dt_{i_1,k_1} then
        Schedule (PU i_1, SU k_1)
        Delete all pairs containing PU i_1 or SU k_1
            from queues 1 and 2
    else
        Schedule [ (PU i_2, PU j_2), (SU k_2, Relay l_2)]
        Delete all pairs containing PU i_2, PU j_2,
            SU k_2, or relay l_2 from queues 1 and 2
    end if
until all PUs or SUs pairs have been scheduled
```

TABLE I

SIMULATION PARAMETERS

Parameters	Values
Cell Radius R	100 m
BS Transmit Power	23 dBm
SU Transmit Power Range	0 – 23 dBm
Relay Node Transmit Power Range	0 – 23 dBm
Maximum Direct Transmission Distance $D_{\max}$	50 m
Number of PUs	16
Number of SU pairs	8
Number of Relay Nodes	200
Noise Power Density	-174 dBm/Hz
Bandwidth BW	180 kHz
Transmission Time Interval (TTI)	1 ms

IV. EXPERIMENTAL RESULTS AND ANALYSIS

A. Parameter Settings

We assume that the BS is located at the center of the cell, while all the PUs, SU pairs and relay nodes are deployed according to a uniform distribution in the cell. The threshold $D_{\max}$ is the maximum distance for DT between any pair of nodes. Given a transmitter and a receiver, the received signal power is $P_R = P_T g_{TR} = P_T |h_{TR}|^2 / d_{TR}^2$, where P_T and P_R are the transmit and received power, respectively, and d_{TR} is the distance. The coefficient h_{TR} follows a complex Gaussian distribution $\mathcal{CN}(0, 1)$. The parameters used in the simulation are summarized in Table I.

B. Performance Study of DF Transmission

We first focus on a single SU pair under DF relaying and make comparisons to DT. The distance between SU transmitter and receiver is assumed to be $D_{\max}$, while they have equal distance to the BS. The relay node is located on the straight line between SU transmitter and receiver. We assume the same group of channel coefficients is used in the simulation. The link capacities are derived from Stackelberg game equilibria and then compared to the link capacity under DT, which is a constant under the above assumptions. However, the link capacity will change along with the position of the relay node. Thus, we first find out the maximum link capacity by setting the relay node on a set of equally spaced locations.

We further define the capacity ratio as the ratio of maximum DF link capacity to DT link capacity. The parameter β is selected from $\{1, 2, 5, 10\}$ and we run 1,000 independent simulations for each β. Since some capacity ratios could be large, we consider the logarithm of the capacity ratios. The cumulative distribution function (CDF) for each β is shown in Fig. 2. When β is relatively small, i.e. $\beta = 1$, the probability to have a lower maximum capacity compared to DT (i.e., the logarithm is below zero) is around 20%. Such a probability will reach 30% and 60% as $\beta = 5, 10$, respectively, meaning that DF transmission is more likely to perform worse than DT when β becomes large. Yet when β grows, DF transmission is more likely to achieve higher transmission rate and result in better *overall* performance in some cases.

Next we study the rate distributions of SU pairs and PUs under different values of β by fixing the relay node at the middle, i.e. $D_{sr,1} = D_{\max}/2$. For $\beta = 1, 2, 5, 10$, we run the simulation 2,000 times under different groups of channel coefficients. The SU link rate distributions are given in Fig. 3. We can see that the follower's relay transmission rate increases as β grows, since the follower is prone to select a higher transmit power under large β as its payment is relatively low.

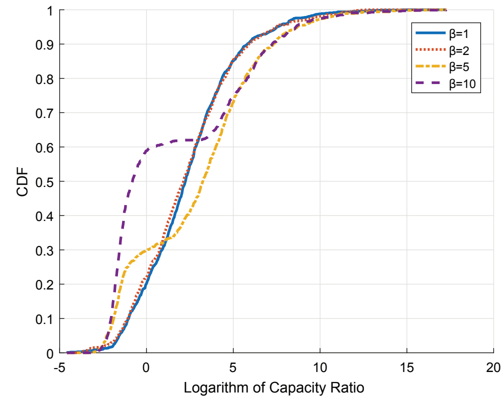

Fig. 2. CDF for logarithm of capacity ratio under different values of β.

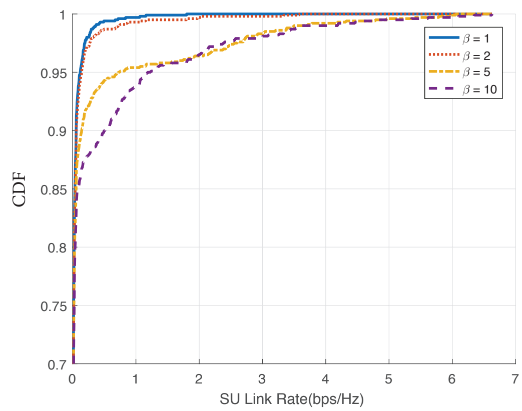

Fig. 3. SU link rate distribution under different β, $D_{sr,1} = D_{\max}/2$.

C. Experimental Study of Scheduling Algorithm

Lastly we study the performance of the proposed hybrid scheduling algorithm. A total of 16 PUs, 8 SU pairs and 200 relay nodes are randomly deployed in the network area, and the number of PUs is large enough to support all transmission. We assume that each SU pair satisfies a maximum distance constraint to adopt DT. Based on the above assumptions, some of the relay nodes could be adopted by multiple SU pairs for DF relay transmission. Once such a relay node is assigned to a certain SU pair, it cannot be adopted by any other SU pairs for DF transmission in the same TTI.

We first study the accumulative SU sum rate in one TTI. The hybrid scheduling and DT scheduling results are given in Fig. 4. It can be seen that DF transmission always achieves higher sum rate compared to DT under the same value of β. As β grows, the follower is likely to buy larger SU and relay transmit power due to their relatively low cost, which

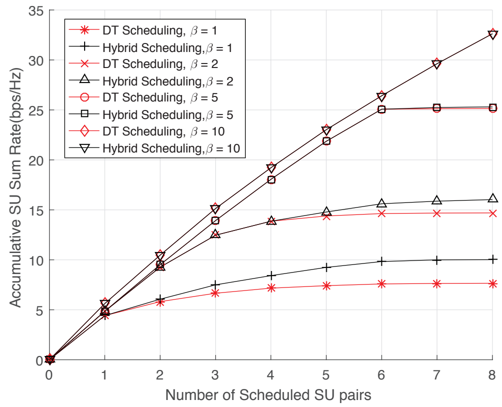

Fig. 4. Accumulative sum rate versus the number of scheduled SU pairs.

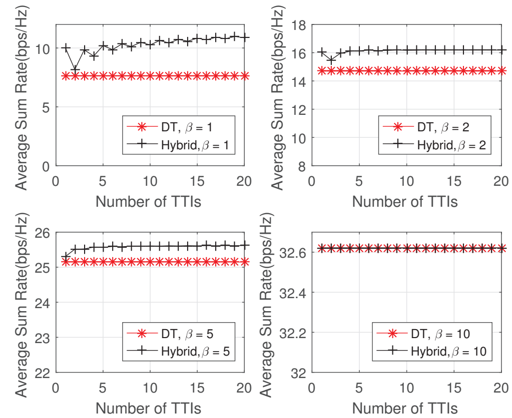

Fig. 5. Average sum rate versus the number of TTIs.

results in a higher SU sum rate. Yet when β is sufficiently large, very little sum rate improvement can be achieved using DF transmission compared to DT scheme. This is due to the fact that when the follower uses higher transmit power, the probability of achieving higher rate in DF transmission significantly decreases, which can be observed in Fig. 2. Since only a small number of relay nodes could be adopted by each SU pair, it is very likely that none of the relay nodes would be scheduled during the process. In this case, the performance of hybrid scheduling is almost the same as that of DT scheduling.

Next we study the average sum rate over a number of consecutive TTIs, where the scheduling algortihm is conducted in each TTI. If two PUs i, j, a SU pair k and a relay node l are scheduled under DF transmission in some TTI, then PU j is preserved and cannot be scheduled in the following TTI as it provides channel for the second hop. Therefore, we can skip all the leader-follower pairs that either contain PU j or relay node l in the two queues in next TTI. Yet SU transmitter k could still use DT or DF transmission as long as it accesses a channel other than channel i. Assume that the maximum number of TTIs is $N = 20$ and we use pipelining as described above in the whole scheduling process. The relationship between the average sum rate and the number of consecutive TTI is given in Fig. 5. Clearly, DT scheduling outcomes is always the same under the same β in each TTI as DT scheduling does not preserve any PUs for the following TTI. As the number of TTIs increases, the average sum rate tends to converge under all values of β and significant improvement is made when β is relatively small. To protect the PUs from harmful SU interference, a small to moderate value of β should be selected.

V. Conclusion

We proposed a novel Stackelberg game framework for SU DF relay transmission in a cognitive radio network. Under the proposed framework, optimal transmission strategies for SU relay pairs were derived. We presented simulation results demonstrating that a significant increase in link capacity can be achieved under the proposed DF relay transmission scheme compared to direct transmission. The simulation results also show that the scale factor β has a significant impact on the achievable link capacity. Lastly, we proposed a hybrid scheduling scheme combining SU direct transmission and DF relay transmission under a Stackelberg game framework, and the experimental results show that good sum rate improvement can be achieved under small to moderate values of β.

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