Gaussian Random Field Approximation for Exclusion Zones in Cognitive Radio Networks

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***Abstract*—To protect primary users from interference caused by secondary users (SUs) in a cognitive radio network, a geographic area called an exclusion zone can be defined in which SUs are prohibited from transmitting using a specified spectrum band. We propose a Gaussian Random Field Model (GRFM) framework for determining an exclusion zone with the desired properties in practical scenarios where analytical specifications may not be available. Based on the GRFM, we derive the radius of a disk determining the exclusion zone, assuming that the SUs are distributed geographically over a planar coverage area. Using measurement data obtained from SUs, the GRFM is applied to approximate the equivalent received signal power and aggregate interference at specified locations. Simulation results show that the GRFM approximation yields an accurate characterization of the exclusion zone.[1]**

I. Introduction

A cognitive radio network consists of primary users (PUs), who have priority to access a licensed spectrum band, and secondary users (SUs), who may utilize the same spectrum under some predefined access rules [1]–[3]. The PUs form a primary system that requires a certain level of communication quality guarantee. The SUs, mostly mobile devices, may constitute a large population within the radio cognitive network. Accordingly, the secondary system generates a high demand for available spectrum resources. Spectrum measurement studies show that spectrum resources are often highly underutilized by the primary system [1]. Dynamic spectrum sharing is seen as an approach to improving spectrum utilization by allowing SUs to sense the spectrum and make use of spectrum holes. Since the activities of SUs may cause interference to the primary system, access rules are needed to allow SUs to dynamically access the spectrum while maintaining the interference to the primary system below a certain threshold [1]–[3].

If the SUs are sufficiently far from the PUs, the interference to the primary system can be maintained at an acceptable level. The areas in which SUs may reuse the spectrum are referred to as spatial holes. From the perspective of the primary system, protection from interference by SUs can be achieved by deactivating SUs when they enter a neighboring region. Such a region is referred to as an *exclusion zone*. The concept of exclusion zone has been widely adopted in scenarios such as ad hoc networks [4], [5] and heterogeneous networks [6]–[8]. In order to be effective, the boundary of the exclusion zone needs to be carefully identified.

Recent approaches to determining exclusion zone boundaries have involved theoretical analysis by fixing the parameter settings of the primary and secondary systems [1]–[4]. Under the assumption of omnidirectional antennas for the primary and secondary transmitters, the exclusion zone has the geometry of a disk [1], [2], [4], [9]–[11]. By applying performance constraints, for example, an aggregate interference constraint or SINR constraint, the radius of the disk can be derived. In practical scenarios, the characteristics of the cognitive radio network are time-varying, such that the exclusion zone is dynamic itself. Thus, alternative measurement-based approaches are needed to effectively characterize the exclusion zone in real-time.

In this paper, we propose a framework for approximating an exclusion zone based on Gaussian Random Field Models (GRFMs). As an efficient tool for characterizing an exclusion zone, GRFMs have the following useful features: 1) the exclusion zone can be considered as a 2-dimensional spatial function, and 2) the SUs can act as spatial samplers. Numerical results for exclusion zones derived from GRFMs constructed from SU samples compare favorably to results derived from the theoretical analysis in [1]. The rest of the paper is organized as follows. In Section II, we describe the assumed system model for a cognitive radio network. In Section III, we develop the GRFM approximation for determining received signal power, aggregate interference, and exclusion zones. In Section IV, we present numerical results for GRFM approximations of received signal power and aggregate interference, along with comparisons to theoretical results from [1]. Concluding remarks are given in Section V.

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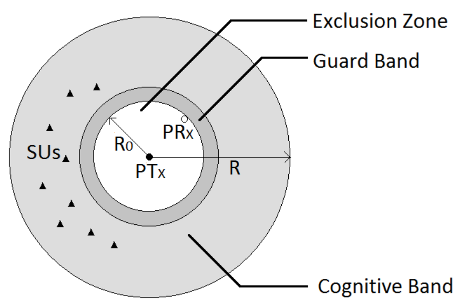

Fig. 1. Geometry of cognitive network.

II. System Model

A. Exclusion Zone and Cognitive Band

We consider a simple model of a cognitive radio network, similar to the model adopted in [1], consisting of a single primary transmitter, at least one primary receiver, and multiple SUs equipped with cognitive radios (see Fig. 1). The primary transmitter is assumed to be located in the center, while the primary receivers are located within a disk of radius R_0 meters, which defines the exclusion zone. Furthermore, SUs should be at least ϵ meters away from any of the primary receivers to ensure that the aggregate interference to these receivers does not become harmful. This *guard band* isolates the exclusion zone from the SUs. The SUs are distributed within an annular area, referred to as the *cognitive band*, with inner radius $R_0 + \epsilon$ and outer radius R.

B. Aggregate Interference

The outage constraint must hold for the worst case in which a primary receiver is located on the border of the exclusion zone and the guard band. This primary receiver is closest to the cognitive band and receives the maximum amount of interference power from the SUs. For simplicity, we assume that the receivers do not adopt multiuser detection. Thus, the aggregate interference to a given primary receiver, denoted as I_0, comes from the transmit power of the SUs. We adopt the simple path-loss channel model used in [1] given by

$$h(d) = \frac{A}{d^{\alpha/2}}, \tag{1}$$

where A is a frequency-dependent amplitude, d is the distance between the transmitter and receiver, and α is the path-loss exponent. For simplicity, we shall normalize A to 1. In addition to path-loss, our system model accounts for the presence of shadowing noise.

We assume that the SUs are uniformly distributed over the geographical area covered by the cognitive radio network, with density λ, while all the SUs may operate concurrently. The network model proposed in [1] assumes that the cognitive band around the exclusion zone is a continuous medium that transmits the signal. Furthermore, the aggregated interference is not defined over the entire cognitive band. However, the cognitive band in a practical network model consists of a finite number of SUs. The aggregate interference received at the primary receiver has the following form:

$$I_0 = \sum_{i=1}^{N} P_{\mathrm{su}} \cdot |h(d_{i,P})|^2, \tag{2}$$

where N is the total number of SUs, P_{su} is the transmit power of each SU, and $d_{i,P}$ is the geographical distance from the ith SU to the primary receiver.

III. GRFM Framework

A. Overview of Gaussian random field model

A GRFM can be viewed as a function predictor that applies a Gaussian random field to determine values of the function at some desired points in the parameter space where the actual function values are unknown or otherwise hard to derive. The inputs of a GRFM are the coordinates of a set of sample points as well as their corresponding function values. Let the function to be predicted be $f(\boldsymbol{x})$ within a d-dimensional parameter space. We would like to predict the function value $f(\boldsymbol{x}^*)$ at a desired point $\boldsymbol{x}^*$. Under the GRFM, the predicted value is modeled as a Gaussian random variable with mean μ and variance σ^2.

Calculation of the predicted value requires the key concept of correlation. Assume we have two arbitrary sample points $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ in the parameter space. Intuitively, $\boldsymbol{x}_i, \boldsymbol{x}_j$ are highly correlated when they are sufficiently close to each other. Therefore, a Gaussian correlation function between $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ is defined as follows:

$$\mathrm{Corr}(\boldsymbol{x}_i, \boldsymbol{x}_j) = \exp\left(-\sum_{l=1}^{d} \theta_l |x_{il} - x_{jl}|^2\right), \tag{3}$$

where θ_l is the correlation parameter in the lth dimension which determines the rate of change in the correlation as $\boldsymbol{x}_i$ approaches $\boldsymbol{x}_j$ in dimension l. If the function parameter space is isotropic, θ_l will be equal to some constant for $l = 1, \ldots, d$; otherwise, the θ_l values will be mutually different. Formulas to determine θ_l can be found in [12].

More generally, given a set of n sample points $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n$ and the corresponding output function values $y_1, \cdots, y_n$, we define the correlation matrix $\boldsymbol{R}$ by

$$\boldsymbol{R}_{ij} = \mathrm{Corr}(\boldsymbol{x}_i, \boldsymbol{x}_j), \; i, j = 1, \cdots, n, \tag{4}$$

where $\boldsymbol{R}_{ij}$ is the (i, j)th entry of matrix $\boldsymbol{R}$. The likelihood function $\mathcal{L}$ of the observed data is given by

$$\mathcal{L} = \frac{1}{[(2\pi\sigma^2)^n |\boldsymbol{R}|]^{1/2}} \exp\left(-\frac{(\boldsymbol{y} - \mu\mathbf{1})^{\mathrm{T}} \boldsymbol{R}^{-1} (\boldsymbol{y} - \mu\mathbf{1})}{2\sigma^2}\right), \tag{5}$$

where $\mathbf{1}$ is a column vector of ones, $\boldsymbol{y} = \mathrm{col}(y_1, \cdots, y_n)$ is the column vector of all outputs, and the superscript $^{\mathrm{T}}$ denotes matrix transpose. By setting its first order derivative to 0, i.e., $\frac{\partial \mathcal{L}}{\partial \mu} = 0$, in maximizing the likelihood function (5), optimal estimates of the output mean and variance can be derived as follows:

$$\hat{\mu} = \frac{\mathbf{1}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{y}}{\mathbf{1}^{\mathrm{T}}\boldsymbol{R}^{-1}\mathbf{1}},\ \hat{\sigma}^2 = \frac{1}{n}\left(\boldsymbol{y} - \mathbf{1}\hat{\mu}\right)^{\mathrm{T}}\boldsymbol{R}^{-1}(\boldsymbol{y} - \mathbf{1}\hat{\mu})). \tag{6}$$

By applying the GRFM, the prediction of the function value $\hat{y}(\boldsymbol{x}^*)$ at the desired point $\boldsymbol{x}^*$ yields

$$\hat{y}(\boldsymbol{x}^*) = \hat{\mu} + \boldsymbol{r}^{\mathrm{T}}\boldsymbol{R}^{-1}(y - \mathbf{1}\hat{\mu}), \tag{7}$$

where $\boldsymbol{r} = \mathrm{col}\left[\mathrm{Corr}(\boldsymbol{x}^*, \boldsymbol{x}_1), \cdots, \mathrm{Corr}(\boldsymbol{x}^*, \boldsymbol{x}_n)\right]$ is a column vector containing Gaussian correlation coefficients between the desired point $\boldsymbol{x}^*$ and all sample points $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n$. Furthermore, the uncertainty of the prediction, denoted by the predicted variance of the function value at $\boldsymbol{x}^*$, is given by

$$\hat{\sigma}^2(\boldsymbol{x}^*) = \hat{\sigma}^2\left[\mathbf{1} - \boldsymbol{r}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{r} + \frac{(\mathbf{1} - \boldsymbol{r}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{r})^2}{\mathbf{1}^{\mathrm{T}}\boldsymbol{R}^{-1}\mathbf{1}}\right]. \tag{8}$$

which is the best linear unbiased predictor [12].

GRFM-based estimation can be regarded as a stochastic learning process using a group of training data from the set of sampling points. The predictive distribution of the function value is Gaussian, i.e., $f(\boldsymbol{x}^*) \sim \mathcal{N}(\hat{y}(\boldsymbol{x}^*), \hat{\sigma^2}(\boldsymbol{x}^*))$. The quality of the model and its computational cost are determined by the training data as well as the distribution of the sample points. Intuitively, a larger number of sampling points results in a more accurate model. However, a larger number of samples requires more computational resources, which may conflict with real-time requirements.

B. Estimation of Exclusion Zone

To estimate the coverage of the exclusion zone, a straightforward way is to investigate its radius under the outage constraint. Since we consider only the shadowing at the receivers and disregard the additive noise, the received SINR is actually the SIR. Therefore, the outage probability constraint of the primary receiver can be expressed as

$$\mathrm{P}(\mathrm{SIR}_{\mathrm{pr}} \leq I_{th}) \leq \beta, \tag{9}$$

where I_{th} is the threshold of SIR, $\mathrm{SIR}_{\mathrm{pr}}$ is the actual SIR at the receiver, and β is a predefined probability threshold, respectively. In theoretical analysis, the received signal power and aggregate interference power can be directly derived from their corresponding closed-form expressions, and the outage probability of the primary receiver can be consequently calculated given I_{th}. By comparing the outage probability with its threshold β, the radius of the exclusion zone can be found.

Suppose that P_r and P_i are, respectively, the received power and aggregate interference at the location of primary receiver in dB scale. Then the probability that SIR is less than or equal to the threshold I_{th} is given by

$$\mathrm{P}(\mathrm{SIR}_{\mathrm{pr}} \leq I_{th}) = \mathrm{P}(P_r - P_i \leq I_{th}), \tag{10}$$

Note that P_r and P_i are both Gaussian variables according to the GRFM assumptions and the SIR is Gaussian as well. The mean and variance estimate for SIR are $\hat{P}_r - \hat{P}_i$ and $\hat{\sigma}_i^2 + \hat{\sigma}_r^2$, respectively, where $\hat{P}_r$, $\hat{P}_i$, $\hat{\sigma}_i^2$, $\hat{\sigma}_r^2$ are the mean and variance estimates for P_r and P_i, respectively. Thus, the probability that the SIR is less than the threshold I_{th} can be rewritten as

$$\mathrm{P}(\mathrm{SIR}_{\mathrm{pr}} \leq I_{th}) = \mathrm{Q}\left(\frac{(\hat{P}_r - \hat{P}_i) - I_{th}}{\sqrt{\hat{\sigma}_i^2 + \hat{\sigma}_r^2}}\right). \tag{11}$$

For any fixed point inside the inner circle of the cognitive band, this probability can be calculated from (11). Intuitively, as the primary receiver gets closer to the center of the network, the SIR it receives increases and eventually exceeds the threshold β. Thus, we select a number of points with various distances from the primary transmitter and approximate their SIR performance. By fixing the outage probability constraint β, the relationship between the exclusion zone radius R_0 and the SIR threshold I_{th} can be derived as

$$I_{th} = \hat{P}_r(R_0) - \hat{P}_i(R_0) - \mathrm{Q}^{-1}(\beta)\sqrt{\hat{\sigma_i}^2(R_0) + \hat{\sigma_r}^2(R_0)}. \tag{12}$$

IV. Numerical Results

In this section, we study the performance of the proposed GRFM through simulation. To effectively characterize the exclusion zone, the equivalent SIR performance at different locations in the network needs to be determined. For the primary receiver, the signal power comes from the transmitter, and the aggregate interference comes from the entire cognitive band. Therefore, we apply GRFM in both these scenarios. To adopt GRFM for approximation, we make use of the MATLAB toolbox DACE [12]. The inputs are the coordinates of the SUs as well as the observation samples they collect. The outputs are the mean and variance estimates of the function value at the desired points.

A. Received Signal Power Approximation

Under the path-loss model in Eq. (1), augmented with lognormal shadowing, the signal power received by SU i from the primary transmitter, in dB scale, is given by

$$P_{R,dB}(i) = P_{T,dB} - 10\alpha\log_{10}(d_i) + W_P, \tag{13}$$

where $P_{T,dB}$ is the power from the primary transmitter in dB, d_i is the distance between primary transmitter and SU i, and the shadowing noise W_P is Gaussian distributed with zero mean and variance σ_S^2. The effect of shadowing noise can be suppressed by measurement

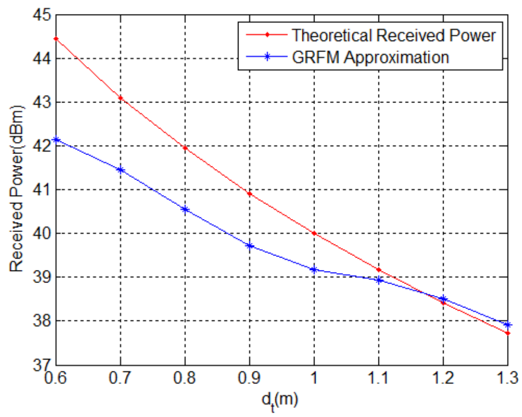

Fig. 2. GRFM approximation for received signal power.

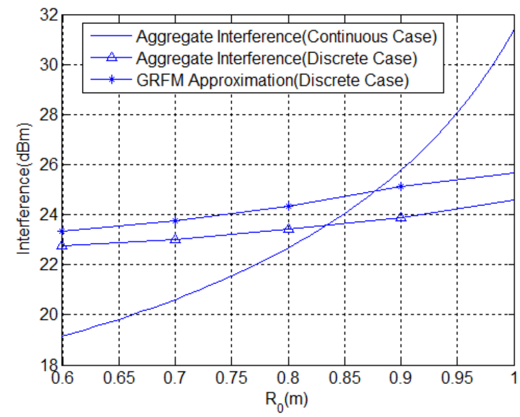

Fig. 3. GRFM approximation for aggregate interference power.

averaging, though it cannot be entirely removed from the received signal. By averaging using M observation samples from a given SU, the variance of the shadowing noise can effectively be reduced by a factor of M, at the expense of additional computational overhead.

In our simulation model, the 2-dimensional propagation plane is first divided into small square grids of equal size $\frac{1}{\sqrt{\lambda}} \times \frac{1}{\sqrt{\lambda}}$ and then SUs are generated randomly within each grid according to a uniform distribution. This approach balances the need to represent both randomness and diversity in the locations of the SUs. If the density of SUs is λ, the width of the grid thus becomes $\frac{1}{\sqrt{\lambda}}$. Once the SUs are generated, the inner and outer bound of the cognitive band are fixed for the simulation. The simulation parameter settings are summarized as follows:

1) Density of cognitive users: $\lambda = 16/\mathrm{m}^2$;
2) Primary transmit power: $P_T = 40$ dBm;
3) Secondary transmit power: $P_{\mathrm{su}} = 0$ dBm;
4) Path loss model: $A = 1$; $\alpha = 2$;
5) Number of selected SUs: $T = 80$;
6) Cognitive band radii: $R_i = 1.1$ m; $R = 5$ m;
7) Number of measurements: $M = 100$;
8) Standard deviation of shadowing: $\sigma_S = 2$ dB.

As discussed in Section II-A, $R_i = R_0 + \epsilon$. The mean number of SUs in the cognitive band is given by $\lambda\pi(R^2 - R_i^2) = 1,196$. Thus, the selected SUs used to obtain the GRFM comprise about $80/1196 \approx 6.7\%$ of the SUs in the cognitive band. The wireless propagation environment is considered to be homogeneous. Therefore, it is reasonable to assume that the weight coefficients in the GRFM satisfy $\theta_1 = \theta_2$. More details on setting GRFM parameters can be found in [12].

To test the performance of the GRFM, we deploy a set of test points in the network with mutually different distances to the primary transmitter, and use the corresponding SUs to approximate the received signal power at these points. Fig. 2 compares the theoretical result from [1] to the GRFM approximation. Along the horizontal axis, d_t [m] denotes the distance between a given desired point and the primary transmitter. Fig. 2 shows the actual power a primary receiver can receive when it is a distance d_t away from the transmitter and its approximated received power if only the power measurements of the SUs are given. When $d_t < 1.1$, it is equivalent to the case that $R_0 = d_t$ and $\epsilon = 1.1 - d_t$. As d_t becomes smaller, the discrepancy between the two curves becomes larger. When d_t is sufficiently small, no SUs are deployed in the vicinity of the desired points, so the GRFM provides a less accurate approximation to the primary transmitter power.

B. Interference Power Approximation

Next, we study the performance of the GRFM in approximating the aggregate interference power to the primary receiver in the presence of shadowing noise. Let N denote the number of SUs. The equivalent aggregate interference power received by SU i is given by

$$P_{I,S}(i) = P_{\mathrm{su}} + \sum_{j=1, j\neq i}^{N} P_I(i,j), \tag{14}$$

where $P_{I,S}(i)$ is the total equivalent aggregate interference power received by SU i, P_{su} is the SU transmit power, and $P_I(i,j)$ is the interference power from SU j, all in linear scale. Similar to (13), the interference power from SU j in dB is given by

$$P_{I,dB}(i,j) = P_{\mathrm{su}} - 10\alpha \log_{10}(d_{ij}) + W_{S,ij}, \tag{15}$$

where d_{ij} is the distance between SUs i and j, and $W_{S,ij}$ represents the lognormal shadowing noise.

The SU transmit power P_{su} is set to 1 mW (0 dBm), and the rest of the GRFM parameter settings for this experiment are the same as in the received signal power approximation in Section IV-A. Under these settings, we repeated the GRFM approximation 1,000 times for each case. The relationship between aggregate interference power at each sample point and the distance to the

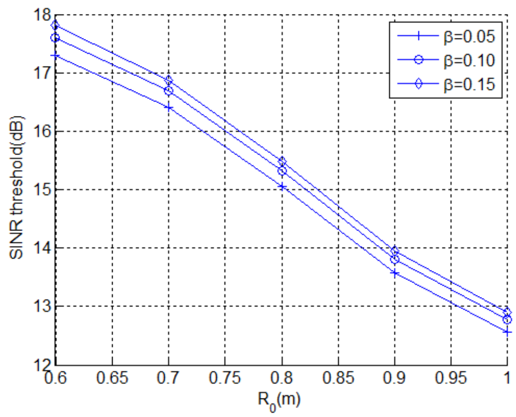

Fig. 4. Relationship between threshold I_{th} and exclusion zone radius R_0.

primary transmitter is shown in Fig. 3. The aggegate interference in the continuous case was obtained using the theoretical analysis proposed in [1] in which the expected interference to the primary receiver is expressed as an integral over the cognitive band. The actual aggregate interference (discrete case) was obtained by summing the transmission power contributions from the finite set of SUs in the cognitive band. It can be observed that GRFM approximation curve approximates the actual aggregate interference curve quite well. When the test point moves to the center of the network, both the GRFM approximation and actual aggregated interference slowly decrease, yet still have an acceptable discrepancy with the theoretical result in the continuous case. This is reasonable because all of the parameters adopted in the continuous and discrete cases are the same. This result demonstrates the effectiveness of the GRFM in estimating the actual aggregate interference distribution.

C. Exclusion Zone Approximation

According to the definition of exclusion zone in Section III-B, the outage probability is calculated through the distribution of received signal power and interference power in the expression of SIR. In the channel model, the approximated aggregated interference $\hat{P}_i$ defined in (11) not only contains the part that comes directly from the interference power approximation in Section IV-B, but also contains the equivalent shadowing that has not been completely removed from the approximated received signal power. Assuming that the approximated interference power is $\hat{P}_{I,S}$ with variance $\hat{\sigma}^2_{I,S}$, the mean value and variance for $\hat{P}_i$ become $\hat{P}_{I,S}$ and $\hat{\sigma}^2_{I,S}+\frac{\sigma^2_S}{M}$, respectively, from properties of Gaussian random variables.

To evaluate the exclusion zone approximation based on the GRFM, we use the same parameters as adopted in Section IV-A. Here, three different values of outage probability constraint β are selected for evaluating the exclusion zone radius R_0, which are 0.05, 0.1 and 0.15, respectively. The relationship between R_0 and the SIR threshold under different outage requirements is shown in Fig. 4. We observe that the SIR threshold decreases to a relative low level (around 13 dB) when the exclusion zone radius extends to $R_0 = 1$, as the harmful interference to the primary receiver significantly increases near the cognitive band. In this experiment, the GRFM approximation is shown to be effective in determining the exclusion zone radius and the relationship between R_0 and other parameters in the proposed network model.

V. Conclusion

We proposed a framework based on GRFMs to approximate the coverage of the exclusion zone in cognitive radio networks using observation samples from the SUs. The negative effect of shadowing can be mitigated by measurement averaging. Our simulation results showed that the GRFM approximation is quite accurate as long as the desired point for prediction is within a reasonable range to the SUs. We assumed a simple disk model of the exclusion zone, but the proposed GRFM framework could be applied to more complicated geometries.

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