

Estimation of Maximum Interference-Free Power Level for Opportunistic Spectrum Access

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Abstract—We consider a scenario in which frequency agile radios opportunistically share a fixed spectrum resource with a set of primary nodes. We develop a collaborative scheme for a group of frequency agile radios to estimate the maximum power at which they can transmit on a given frequency channel, without causing harmful interference to the primary receivers. The proposed scheme relies on signal strength measurements taken by a group of frequency agile radios, which are then used by a target node to characterize the spatial size of its perceived spectrum hole in terms of the maximum permissible transmit power. We derive an approximation to the maximum interference-free transmit power using the Cramér-Rao bound on localization accuracy. We present numerical results to demonstrate the effectiveness of the proposed scheme under a variety of scenarios.

Index Terms—Spectrum sharing, cognitive radio, radio resource management, geolocation, Cramér-Rao bound.

I. INTRODUCTION

IN conventional wireless systems, the spectrum is allocated statically among a set of transmitters over a geographic coverage area. Recent studies have shown that significant portions of the wireless spectrum are highly underutilized [2], [3]. In principle, such “spectrum holes” could be exploited by frequency-agile radios (FARs), which are capable of dynamically tuning to different frequency ranges. Frequency agility and high receiver sensitivity are key features of emerging cognitive radios (CRs) [4]–[6]. A group of FARs could exploit the presence of spectrum holes in the allocated spectrum by communicating on frequency channels lying within the holes. An open research question is whether effective opportunistic spectrum sharing can be realized efficiently and practically. Some recent research work in this area can be found in [7].

In this paper, we focus on the problem of estimating the size of a spectrum hole in terms of the maximum power that a FAR node can transmit on a given frequency channel without causing harmful interference to primary users. In [8], the impact of secondary transmissions on a primary receiver is studied in terms of interference probability. Because of the integral forms involved it is computationally difficult to solve for the allowable secondary transmit power. In [9], [10], an additional *no-talk radius* is defined within which the secondary

users must be quiet to guarantee service to primary users within a *protected radius*. Once these distances are specified (in terms of SNR margins), the aggregate interference at the edge of the protected region is computed, which can then be used to obtain the total permissible secondary transmit power. However, this approach assumes that the primary transmit power and the local SNR at the secondary receivers are already accurately known, so that SNR can be used as a proxy for distance. To avoid these limitations, our approach exploits collaboration among secondary nodes for explicit sensing of the primary transmitter’s power and location. The advantage of utilizing location information via collective sensing is discussed in [11]–[13] and the importance of exploiting spatial statistics to characterize the cognitive radio environment is studied in [14], [15].

A basic mechanism for opportunistic spectrum access is the Listen-Before-Talk (LBT) scheme [16]. In the LBT scheme, a FAR node “listens” on a given frequency channel. When the channel is sensed idle, the FAR node has the opportunity to “talk,” i.e., to transmit on the channel for up to a certain maximum duration at a power level not exceeding a fixed threshold. To avoid causing harmful interference to the primary users, the maximum power level and the maximum talk duration in LBT must be chosen relatively conservatively in practice. This can severely limit the potential capacity gains that could be achievable with opportunistic spectrum sharing.

Higher capacity gains could be achieved if the FAR nodes were capable of collaborating and exchanging local information concerning the primary user’s transmission characteristics. In [16], a simple collaborative version of LBT was shown to improve the spectrum sharing capacity gain by an order of magnitude. To further improve effectiveness of opportunistic spectrum sharing, signal strength (SS) measurements of the primary user could be shared by FAR nodes and used by a given FAR node to determine the maximum power level at which it can transmit without causing harmful interference to the primary user. We refer to this power level as the maximum interference-free transmit power (MIFTP).

We develop a method to estimate the MIFTP for a given FAR node on a given frequency channel, based on SS measurements collected by one or more FAR nodes in the vicinity of a primary transmitter. The MIFTP characterizes the size of the spectrum hole in the spatial domain with respect to a given FAR node and frequency channel. The SS measurements may be obtained by a single FAR node at different locations at different points in time, or by collaborative sharing of measurement information among spatially separated FAR nodes. We assume that the primary transmitter transmits at

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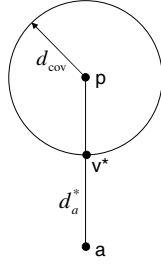


Fig. 1. FAR node a outside coverage area of primary transmitter p .

constant power during an observation period. Thus, we do not address the separate issue of opportunistic spectrum access in the time-domain, i.e., exploiting periods for which the primary transmitter may be idle [17], [18].

Our proposed approximation for MIFTP is derived from the maximum likelihood estimate (MLE) of the location of the primary transmitter based on SS measurements and the associated Cramér-Rao bound (CRB) on the error of the estimator. The primary transmitter power is estimated as an additional parameter and incorporated into the MIFTP approximation. The MIFTP specifies the range for power control algorithms employed by secondary users. Such a power control scheme is proposed in [19], under the assumption that the primary system's location and transmit power are known without error. In [20], a method for determining the MIFTP is proposed whereby the distance between the primary and secondary transmitters is inferred using spectrum sensing decisions and then the results of [9] are used to find the maximum transmit power for the secondary user. While this approach avoids explicit localization of the primary transmitter, it requires knowledge of the path loss function and the primary's transmit power.

The remainder of the paper is organized as follows. Section II discusses the key concepts of opportunistic spectrum sharing and MIFTP. Section III describes the canonical SS localization model and extends it to the case where the transmit power is unknown. Section IV derives an approximation for the MIFTP. Section V presents numerical results, which demonstrate the effectiveness of the proposed approach to spectrum sharing. Finally, the paper is concluded in Section VI.

II. MAXIMUM INTERFERENCE-FREE TRANSMIT POWER

Consider a FAR node a and a primary transmitter p , which transmits on a given frequency channel γ . We define a spectrum hole with respect to γ for the FAR node a in terms of the maximum power at which the node a can transmit without causing harmful interference to any potential primary receiver or victim node v , within the range of the primary transmitter p . This maximum power level is called the MIFTP and is defined more precisely as the maximum transmit power of node a on a target frequency channel γ , such that the probability of interference to any victim node v is less than a prescribed threshold (cf. [16]).

We shall assume that all transmissions are omnidirectional¹ and the signal propagation is governed by a lognormal shadow-

ing model (cf. [21]). Hence, the propagation loss in dB between two nodes i and j can be expressed as

$$L_{i,j} = g(d_{i,j}, \epsilon_{i,j}) + W \text{ [dB]}, \quad (1)$$

where the function $g(d, \epsilon)$, represents the path loss component, with ϵ denoting the path loss factor. Although in practice, $\epsilon_{i,j}$ depends on the specific propagation condition between nodes i and j (for example, line-of-sight versus non-line-of-sight, indoor versus outdoor, urban versus rural, etc.), throughout this work we shall assume that $\epsilon_{i,j}$ is a fixed known constant, i.e., $\epsilon_{i,j} = \epsilon$. For simplicity, we assume that $g(d, \epsilon) = 10\epsilon \log_{10} d$ and denote it by $g(d)$. More complicated path loss models could be incorporated into our analysis, such as the empirical propagation model (EPM-73) [22], Longley-Rice model [23], or the TIREM (Terrain Integrated Rough Earth Model) [24]. We assume that the shadowing noise $W \sim \mathcal{N}(0, \sigma_W^2)$, where σ_W has units of dB. The received power at node v due to node p is

$$R_v = s_p - L_{p,v} = s_p - g(d_{p,v}) + W \text{ [dBm]}, \quad (2)$$

where s_p is the transmit power of node p . The received power at node v from node a is given by

$$I_v = s_a - L_{a,v} = s_a - g(d_{a,v}) + W \text{ [dBm]}, \quad (3)$$

where s_a is the transmit power of a .

The *outage probability* of a victim node v with respect to the transmitter p , is the probability that the received power R_v from node p is below a detection threshold r_{\min} [dBm]:

$$P_{\text{out}}(p, v) \triangleq P\{R_v < r_{\min}\}, \quad (4)$$

when p is transmitting. In general, r_{\min} is determined by the primary receiver's structure, noise statistics and QoS. The *coverage distance* is the maximum distance between the node p and any potential victim node v such that the outage probability does not exceed a threshold $\epsilon_{\text{cov}} > 0$:

$$d_{\text{cov}}(p) \triangleq \max\{d_{p,v} : P_{\text{out}}(p, v) \leq \epsilon_{\text{cov}}\} \\ = g^{-1}(s_p - r_{\min} + \sigma_W Q^{-1}(1 - \epsilon_{\text{cov}})), \quad (5)$$

where $g^{-1}(\cdot)$ denotes the inverse of $g(\cdot)$ and $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$ denotes the standard Q -function. Note that $d_{\text{cov}}(p)$ depends on s_p , r_{\min} , ϵ_{cov} , σ_W^2 and the path loss function $g(\cdot)$. We assume that the FAR node knows or can estimate s_p and therefore can evaluate $d_{\text{cov}}(p)$. The circle centered at node p with radius $d_{\text{cov}}(p)$ is the *coverage area* of the transmitter p . Any potential victim node v , which lies outside of coverage area of node p would be oblivious to the interference caused by the FAR node a . The *interference probability* with respect to a given victim node v is the probability that I_v exceeds an interference tolerance threshold i_{\max} [dBm]:

$$P_{\text{int}}(a, v) \triangleq \Pr\{I_v \geq i_{\max}\}, \quad (6)$$

when node a is transmitting. This threshold can be set to meet the primary system's interference tolerance policy. Under the shadowing model (1), the interference probability is given by

$$P_{\text{int}}(a, v) = Q\left(\frac{i_{\max} - s_a + g(d_{a,v})}{\sigma_W}\right). \quad (7)$$

¹See Section III.D for a brief discussion of the case of directional radiation.

For a fixed primary transmitter p and FAR node a , the MIFTP is the maximum transmit power of the FAR node such that the interference probability with respect to any potential victim node within the coverage distance from node p does not exceed a threshold $\varepsilon_{\text{int}} > 0$:

$$s_a^* \triangleq \max\{s_a : P_{\text{int}}(a, v) \leq \varepsilon_{\text{int}}, \forall v : d_{p,v} \leq d_{\text{cov}}(p)\} \text{ [dBm]}. \quad (8)$$

Alternatively, the MIFTP can be defined in terms of the worst-case interference probability:

$$P_{\text{int}}(a) = \sup_v P_{\text{int}}(a, v) = Q\left(\frac{i_{\text{max}} - s_a + g(d_a^*)}{\sigma_W}\right), \quad (9)$$

where $d_a^* \triangleq d_{p,a} - d_{\text{cov}}(p)$ is called the *critical distance* for the FAR node a with respect to the primary transmitter p and the supremum is taken over all potential victim nodes v such that $d_{p,v} \leq d_{\text{cov}}(p)$. Then $s_a^* = \max\{s_a : P_{\text{int}}(a) \leq \varepsilon_{\text{int}}\}$.

Proposition 1: The MIFTP is given by²

$$s_a^* = \begin{cases} i_{\text{max}} + g(d_a^*) - \sigma_W Q^{-1}(\varepsilon_{\text{int}}), & \text{if } d_{p,a} > d_{\text{cov}}(p), \\ -\infty, & \text{otherwise.} \end{cases} \quad (10)$$

Proof: If $d_{p,a} \leq d_{\text{cov}}$, a victim node v can be placed arbitrarily close to node a within the coverage area. Hence, node a cannot transmit without potentially causing interference to a victim node, i.e., $s_a^* = -\infty$ [dBm] in this case. If node a lies outside the coverage area (see Fig. 1), then $d_{p,a} > d_{\text{cov}}(p)$. In this case, the minimum distance to a potential victim node lying within the coverage area is given by $d_a^* = d_{p,a} - d_{\text{cov}}(p)$. To avoid causing harmful interference to victim nodes lying within the coverage area, condition (8) must be satisfied. Using (3), this implies that $s_a \leq i_{\text{max}} + g(d_a^*) - \sigma_W Q^{-1}(\varepsilon_{\text{int}})$. ■

III. ROLE OF LOCALIZATION AND CRB

Localization in the context of spectrum hole discovery differs from more conventional scenarios (cf. [25]) in two respects: (1) The FAR nodes collaboratively localize the primary transmitter. (2) No cooperation is assumed between the FAR node and the primary transmitter. It is assumed that the FAR nodes know their own locations via GPS (Global Positioning System) or some type of self-localization scheme (cf. [26]–[29]). Signal strength (SS) information provides the simplest and most natural means of localization.

A. SS-based localization

Let $\mathbf{L} = [x_p, y_p]^T$ denote the location of the primary transmitter. Now suppose that a set of uncorrelated SS measurements, $\{S_1, \dots, S_N\}$, is available, together with a corresponding set of position coordinates $\{\mathbf{L}_1, \dots, \mathbf{L}_N\}$, where $\mathbf{L}_i = [x_i, y_i]^T$, $i = 1, \dots, N$. The set of observables, $\mathcal{O} \triangleq \{(S_i, \mathbf{L}_i) : i = 1, \dots, N\}$, may be obtained in several ways. For example, consider a scenario in which N FAR nodes, located at positions $\mathbf{L}_1, \dots, \mathbf{L}_N$, collect the signal strength observables S_1, \dots, S_N at a given time. The FAR nodes exchange their observables among each other, such that

²When $s_a^* = -\infty$, the FAR node should not attempt to transmit on the target frequency channel.

at least one of the FAR nodes receives the entire set \mathcal{O} . Such a FAR node can then compute an estimate $\hat{\mathbf{L}} = [\hat{X}_p, \hat{Y}_p]^T$ of the location of the primary transmitter. Alternatively, the observable set \mathcal{O} may be obtained by measurements from a single FAR node at N different points in time along a trajectory as the node moves in the coverage area. In general, a given observable (S_i, \mathbf{L}_i) may be obtained either from a measurement taken by the FAR node itself in the past, or from a measurement by another FAR node that shares this information.

Given a set of observations, \mathcal{O} , the observation equations can be written as

$$\mathbf{S} = \mathbf{z} + \mathbf{W}, \quad (11)$$

where $\mathbf{S} = [S_1, \dots, S_N]^T$, $\mathbf{z} = [z_1, \dots, z_N]^T$, and $\mathbf{W} = [W_1, \dots, W_N]^T$, with $z_i = s_p - g(d_i)$, $d_i = \sqrt{(x_i - x_p)^2 + (y_i - y_p)^2}$ and W_i denoting the shadowing noise component. An estimate $\hat{\mathbf{L}}$ of the location of the primary transmitter can be obtained from the SS observation equation (11). Given N uncorrelated observations the *likelihood function* is

$$f_{S|\mathbf{L}}(\mathbf{S}) = \frac{1}{(2\pi)^{\frac{N}{2}} |\mathbf{\Lambda}|^{\frac{1}{2}}} \exp\left\{-\frac{1}{2}(\mathbf{S} - \mathbf{z})^T \mathbf{\Lambda}^{-1}(\mathbf{S} - \mathbf{z})\right\}, \quad (12)$$

where $\mathbf{\Lambda} = \sigma_W^2 \mathbf{I}$, \mathbf{I} is the $N \times N$ identity matrix and $|\mathbf{\Lambda}|$ denotes the determinant of $\mathbf{\Lambda}$. The ML estimate of \mathbf{L} is determined by solving the following nonlinear optimization problem: $\hat{\mathbf{L}}_{\text{ML}} = \arg \max_{\mathbf{L}} f_{S|\mathbf{L}}(\mathbf{S})$.

B. Cramér-Rao lower Bound

The Cramér-Rao lower bound (CRB) provides a lower bound on the variance (or covariance matrix) of any unbiased estimate of an unknown parameter. For the SS localization model in (11), the CRB of any unbiased estimate $\hat{\mathbf{L}}$ of \mathbf{L} is given by

$$E_{\mathbf{L}}[(\hat{\mathbf{L}} - \mathbf{L})(\hat{\mathbf{L}} - \mathbf{L})^T] \geq \mathbf{J}_{\mathbf{L}}^{-1}, \quad (13)$$

where $E_{\mathbf{L}}[\cdot]$ denotes conditional expectation with respect to \mathbf{L} and $\mathbf{J}_{\mathbf{L}}$ is the Fisher information matrix (FIM) given by

$$\mathbf{J}_{\mathbf{L}} = E_{\mathbf{L}} \left[\frac{\partial}{\partial \mathbf{L}} \ln f_{S|\mathbf{L}}(\mathbf{S}) \left(\frac{\partial}{\partial \mathbf{L}} \ln f_{S|\mathbf{L}}(\mathbf{S}) \right)^T \right],$$

where $f_{S|\mathbf{L}}(\mathbf{S})$ is the likelihood function. In (13), the matrix inequality $\mathbf{A} \geq \mathbf{B}$ should be interpreted as the assertion that the matrix $\mathbf{A} - \mathbf{B}$ is non-negative definite. The CRB provides a lower bound on the mean-squared errors for the components of \mathbf{L} .

If the primary transmitter's signal power is known, the FIM can be expressed as follows [25]:

$$\mathbf{J}_{\mathbf{L}} = \left(\frac{10\epsilon}{\sigma_W \ln 10} \right)^2 \mathbf{H} \mathbf{D}^2 \mathbf{H}^T, \quad (14)$$

where

$$\mathbf{H} \triangleq \begin{bmatrix} \cos \phi_1 & \cos \phi_2 & \cdots & \cos \phi_N \\ \sin \phi_1 & \sin \phi_2 & \cdots & \sin \phi_N \end{bmatrix}, \quad \mathbf{D} \triangleq \text{diag} [d_1^{-1}, \dots, d_N^{-1}], \quad (15)$$

and $\phi_i = \tan^{-1} \left(\frac{y_p - y_i}{x_p - x_i} \right)$, $i = 1, \dots, N$ is the angle between the x -axis and the line connecting (x_i, y_i) and (x_p, y_p) measured counterclockwise. It can be shown (cf. [25], [30]) that the CRB is achieved by the maximum likelihood estimate (MLE) asymptotically as $\sigma_W^2 \rightarrow 0$.

C. Unknown Transmitter Power

For opportunistic spectrum access, the transmit power is unknown. Therefore, the parameter vector to be estimated and its MLE are given by $\Theta = [x_p, y_p, s_p]^T$ and $\hat{\Theta}_{\text{ML}} = [\hat{X}_p, \hat{Y}_p, \hat{S}_p]^T$, respectively. The next proposition gives a closed-form expression for the CRB and its achievability condition when the transmit power is unknown (see Appendix A for a proof).

Proposition 2: The CRB is given by

$$\mathbf{J}_{\Theta}^{-1} = \begin{bmatrix} \mathbf{J}_{\mathbf{L}}^{-1} + b^{-1} \mathbf{c} \mathbf{c}^T & -b^{-1} \mathbf{c} \\ -b^{-1} \mathbf{c}^T & b^{-1} \end{bmatrix}, \quad (16)$$

where

$$b \triangleq \frac{N}{\sigma_W^2} - \mathbf{a}^T \mathbf{J}_{\mathbf{L}}^{-1} \mathbf{a}, \quad \mathbf{c} \triangleq \mathbf{J}_{\mathbf{L}}^{-1} \mathbf{a},$$

$$\mathbf{a} \triangleq -\frac{10\epsilon}{\sigma_W^2 \ln 10} \left[\sum_{i=1}^N \frac{\cos \phi_i}{d_i}, \sum_{i=1}^N \frac{\sin \phi_i}{d_i} \right]^T, \quad (17)$$

assuming that $\mathbf{J}_{\mathbf{L}}$ given in (14) is invertible. The CRB corresponding to (16) is achievable by the MLE as the observation noise becomes vanishingly small, i.e., $\sigma_W \rightarrow 0$.

It can be shown that when s_p is unknown, the *likelihood function* $f_{\mathcal{S}|\Theta}(\mathcal{S})$ is:

$$f_{\mathcal{S}|\Theta}(\mathcal{S}) = \frac{1}{(2\pi)^{\frac{N}{2}} |\mathbf{\Lambda}|^{\frac{1}{2}}} \exp \left\{ -\frac{1}{2} (\mathcal{S} - \mathbf{z})^T \mathbf{\Lambda}^{-1} (\mathcal{S} - \mathbf{z}) \right\}. \quad (18)$$

The ML estimate of Θ is determined by solving the following nonlinear optimization problem: $\hat{\Theta}_{\text{ML}} = \arg \max_{\Theta} f_{\mathcal{S}|\Theta}(\mathcal{S})$.

D. Generalization to directional transmission

The estimation problem changes for a directional radiation pattern of the primary transmitter. As a first step, assume that the side lobe leakage is negligible and all signal strength measurements are taken by nodes located within the main lobe of the primary's radiation pattern. Let f denote primary antenna's beam pattern function, which characterizes the variation of the antenna's power pattern in the main lobe as a function of angle. Assuming that the form of f is known, the parameters that define it must be estimated in addition to the other parameters. Then the i^{th} received power (in dB scale) is given by

$$S_i = s_p + 10 \log_{10} f_i^2(\mathbf{p}) - g(d_i) + W_i \text{ [dBm]}, \quad (19)$$

where \mathbf{p} denotes a vector of parameters that characterize f . If the form of f is unknown, it can be approximated by a window-like function with an appropriate roll-off. In such a scenario, it may be necessary to employ array processing in some of the FAR nodes. The estimation of \mathbf{p} will increase the error variance of the other parameters due to the nuisance parameter effect. In general, more measurements would be needed to offset the increase in error due to the estimation of \mathbf{p} . A detailed treatment of this issue is beyond the scope of the present paper.

IV. APPROXIMATION FOR MIFTP

The true MIFTP, as given in Proposition 1, cannot be calculated directly, since the true location, $\mathbf{L} = [x_p, y_p]^T$, of the primary transmitter p is unknown. In this section, we derive an approximation for the MIFTP when the transmit power is known as well as unknown.

A. Known transmit power

Assume first that the transmit power s_p of the primary transmitter is a known constant. Let $\hat{\mathbf{L}}_{\text{ML}} = [\hat{X}_p, \hat{Y}_p]^T$ denote the MLE of \mathbf{L} . Given a set of $N \geq 3$ independent SS measurements from the primary transmitter, obtained by the FAR nodes, $\hat{\mathbf{L}}_{\text{ML}}$ provides an unbiased estimate of \mathbf{L} as the shadowing noise tends to zero; i.e., $\hat{\mathbf{L}}_{\text{ML}}$ is asymptotically *efficient* as $\sigma_W^2 \rightarrow 0$. As discussed in Section III-B, in this asymptotic regime, the mean squared error of $\hat{\mathbf{L}}_{\text{ML}}$ achieves the CRB, which we denote by $\mathbf{J}_{\mathbf{L}}^{-1}$. Suppose that the FAR node a is located at $\mathbf{L}_a = [x_a, y_a]^T$. Given $\hat{\mathbf{L}}_{\text{ML}}$, the MLE for the distance $d_{p,a}$, denoted by $\hat{D}_{p,a}$, can be obtained by applying the invariance principle (cf. [31], p. 217), which states that the MLE of a function $h(\cdot)$ of \mathbf{L} is given by $h(\hat{\mathbf{L}})$, where $\hat{\mathbf{L}}$ denotes the MLE of \mathbf{L} . Hence, we obtain

$$\hat{D}_{p,a} = \sqrt{(\hat{X}_p - x_a)^2 + (\hat{Y}_p - y_a)^2}.$$

Proposition 3: In the asymptotic regime $\sigma_W^2 \rightarrow 0$, the MLE $\hat{D}_{p,a}$ achieves the associated CRB, given by $J_{p,a}^{-1} \triangleq \mathbf{H}_{p,a}^T \mathbf{J}_{\mathbf{L}}^{-1} \mathbf{H}_{p,a}$, where

$$\mathbf{H}_{p,a} \triangleq [\cos \phi_{p,a}, \sin \phi_{p,a}]^T, \quad \phi_{p,a} = \tan^{-1} \left(\frac{y_p - y_a}{x_p - x_a} \right).$$

A proof is given in Appendix B.

Let $E_{p,a} \triangleq \hat{D}_{p,a} - d_{p,a}$ denote the estimation error of $\hat{D}_{p,a}$. Proposition 3 implies that in the asymptotic regime $\sigma_W^2 \rightarrow 0$, $E_{p,a}$ is Gaussian with zero mean and variance $J_{p,a}^{-1}$ (cf. [32, Appendix D]), i.e., $E_{p,a} \sim \mathcal{N}(0, J_{p,a}^{-1})$. Define $\beta \triangleq \hat{D}_{p,a} - d_{\text{cov}}(p)$. Suppose $E_{p,a} = r$. If $|r| \geq \beta > 0$, then in the worst case, the FAR node lies within $d_{\text{cov}}(p)$ of the true primary transmitter p (see Fig. 2(a)). In this scenario, the FAR node must not transmit, i.e., $s_a^* = -\infty$, to avoid potentially harmful interference to the victim nodes. If $0 < |r| < \beta$, then the FAR can transmit, i.e., $s_a^* \neq -\infty$ (see Fig. 2(b)).

Proposition 4: Under the Gaussian assumption for $E_{p,a}$ and for $|r| \leq 0.993\beta$, the interference probability conditioned on $E_{p,a}$ is upper bounded as follows:

$$P_{\text{int}}(a, v | E_{p,a} = r) \leq Q(b_1 + b_2 |r|), \quad (20)$$

where

$$b_1 \triangleq \frac{i_{\text{max}} + 10\epsilon \log_{10} \beta - s_a}{\sigma_W}, \quad b_2 \triangleq -\frac{50\epsilon}{\beta \sigma_W \ln 10}. \quad (21)$$

A proof is given in Appendix C.

Requiring that $s_a \neq -\infty$, we obtain

$$P_{\text{int}}(a, v) = \int_{-\beta}^{\beta} P_{\text{int}}(a, v | E_{p,a} = r) f_{E_{p,a}}(r) dr,$$

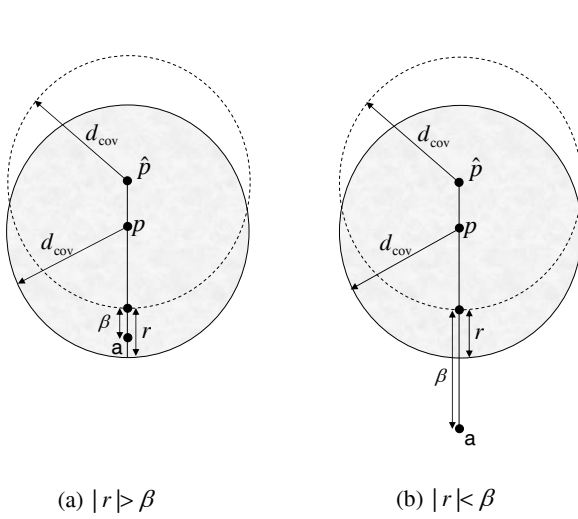


Fig. 2. Illustration of relationship between r and β . If $0 < |r| < \beta$, $s_a^* \neq -\infty$. Otherwise, $s_a^* = -\infty$.

where $f_{E_{p,a}}(r)$ denotes the probability density function (pdf) of $E_{p,a}$. Using (20) and a result from [33, p. 102], we obtain

$$P_{\text{int}}(a, v) \leq Q \left(\frac{b_1}{\sqrt{1 + b_2^2 J_{p,a}^{-1}}} \right). \quad (22)$$

To obtain an expression for the MIFTP, we require the FAR node transmit power, s_a , to be less than or equal to the right-hand side of (22), which implies

$$s_a \leq i_{\max} + 10\epsilon \log_{10} \beta - c(\sigma_W, \epsilon, \beta, J_{p,a}, \epsilon_{\text{int}}), \quad (23)$$

where

$$c(\sigma, \epsilon, \beta, J, \epsilon) \triangleq \sigma \sqrt{1 + \left(\frac{50\epsilon}{\beta \sigma \ln 10} \right)^2 J^{-1}} \cdot Q^{-1}(\epsilon). \quad (24)$$

The right-hand side of (23) provides an approximation for the MIFTP, but since the true CRB of $\hat{D}_{p,a}$, i.e., $J_{p,a}^{-1}$ is unknown, we replace it with the MLE of $J_{p,a}^{-1}$, which is denoted by $\hat{J}_{p,a}^{-1}$. This is justified by the invariance principle mentioned earlier and validated by our numerical studies (see Section V).

Recall that for $s_a^* \neq -\infty$, we require that a particular realization of the random variable $E_{p,a} = r$, satisfy the worst case scenario $0 < |r| < \beta$. Since we do not know r , we can only ensure that for $\beta > 0$, the event $(|E_{p,a}| < \beta)$ is satisfied with high probability. For $s_a^* \neq -\infty$ and $\epsilon > 0$ (close to 1), we require $\beta > \beta^* > 0$, where

$$\beta^* \triangleq \min \left\{ \tilde{\beta} : \Pr(|E_{p,a}| < \tilde{\beta}) \geq \epsilon \right\} = \sqrt{J_{p,a}^{-1}} Q^{-1} \left(\frac{1-\epsilon}{2} \right).$$

For example, for $\epsilon = 0.9973$, $\beta^* \approx 3\sqrt{J_{p,a}^{-1}}$. Again as before, we replace $J_{p,a}^{-1}$ by $\hat{J}_{p,a}^{-1}$, i.e., we have $\hat{\beta}^* = \sqrt{\hat{J}_{p,a}^{-1}} \cdot Q^{-1} \left(\frac{1-\epsilon}{2} \right)$. Hence, we obtain the following approximation for the MIFTP:

$$\hat{s}_a = i_{\max} + 10\epsilon \log_{10} \beta - c(\sigma_W, \epsilon, \beta, \hat{J}_{p,a}, \epsilon_{\text{int}}), \quad (25)$$

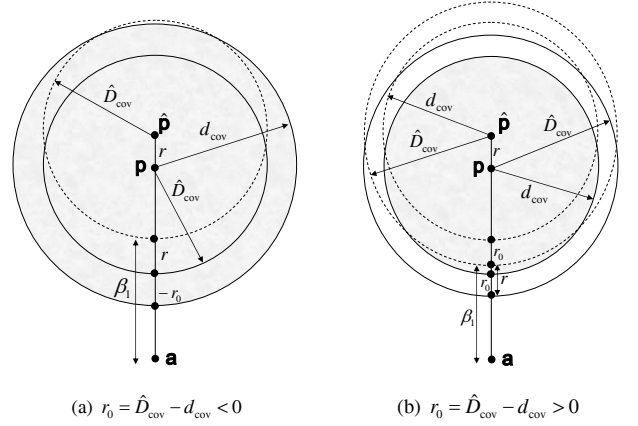


Fig. 3. Illustration of relationship between r , r_0 , and β . In both cases, $0 < |r - r_0| < \beta_1$, which implies that $s_a^* \neq -\infty$.

if and only if $\beta > \hat{\beta}^* > 0$. We point out that as the accuracy of the estimate $\hat{D}_{p,a}$ improves, the CRB estimate $\hat{J}_{p,a}^{-1}$ tends to zero and the right-hand side of (25) converges to the true MIFTP as given in (10). The approximate formula (25) for MIFTP requires at least three independent SS measurements, i.e., $N \geq 3$, which should be obtained from FAR nodes in the vicinity of the primary transmitter.

B. Unknown transmit power

If the primary transmitter power s_p is unknown, it can be estimated together with the location \mathbf{L} as a parameter vector $\Theta = [\mathbf{L}, s_p]$, as discussed in Section III-C. From that section, we know that the MLE, $\hat{\Theta}_{\text{ML}}$, achieves the CRB asymptotically as $\sigma_W^2 \rightarrow 0$. Note that, since $d_{\text{cov}}(p)$ depends on s_p (cf. (5)), an estimate of $d_{\text{cov}}(p)$ is needed as well. Hence it is convenient to work in terms of $\tilde{\Theta} \triangleq [d_{p,a} \ d_{\text{cov}}(p)]^T$ and its associated CRB $\mathbf{J}_{\tilde{\Theta}}^{-1}$, instead of Θ and its associated CRB \mathbf{J}_{Θ}^{-1} . Invoking the invariance principle again, we have $\hat{\Theta}_{\text{ML}} = [\hat{D}_{p,a} \ \hat{D}_{\text{cov}}(p)]^T$, where $\hat{D}_{p,a} = \sqrt{(\hat{X}_p - x_a)^2 + (\hat{Y}_p - y_a)^2}$ and $\hat{D}_{\text{cov}}(p) = g^{-1}(\hat{S}_p - r_{\min} + \sigma_W Q^{-1}(1 - \epsilon_{\text{cov}}))$. Define $E_1 \triangleq E_{p,a} - E_{\text{cov}}$, where $E_{p,a} \triangleq \hat{D}_{p,a} - d_{p,a}$ and $E_{\text{cov}} \triangleq \hat{D}_{\text{cov}}(p) - d_{\text{cov}}(p)$.

Proposition 5: In the asymptotic regime $\sigma_W \rightarrow 0$, E_1 can be modeled as $E_1 \sim \mathcal{N}(0, J_1^{-1})$, where $J_1^{-1} \triangleq \text{Tr}(\mathbf{J}_{\tilde{\Theta}}^{-1}) - 2[\mathbf{J}_{\tilde{\Theta}}^{-1}]_{(1,2)}$ and $\mathbf{J}_{\tilde{\Theta}}^{-1} = \mathbf{H}_1^T \mathbf{J}_{\Theta}^{-1} \mathbf{H}_1$, where

$$\mathbf{H}_1 \triangleq \begin{bmatrix} \cos \phi_{p,a} & \sin \phi_{p,a} & 0 \\ 0 & 0 & \frac{\ln 10}{10\epsilon} d_{\text{cov}}(p) \end{bmatrix}^T.$$

A proof can be found in [32].

Let $E_{p,a} = r$, $E_{\text{cov}} = r_0$, and $\beta_1 = \hat{D}_{p,a} - \hat{D}_{\text{cov}}(p)$. The FAR node can transmit with positive power, i.e., $s_a^* \neq -\infty$, if $0 < |r - r_0| < \beta_1$, otherwise $s_a^* = -\infty$ (see Fig. 3). Here, the critical distance from the FAR node is given by

$$d_a^* = d_{p,a} - d_{\text{cov}}(p) = \hat{D}_{p,a} - r - (\hat{D}_{\text{cov}}(p) - r_0) = \beta_1 - r_1,$$

where $r_1 \triangleq r - r_0$. We note that here r_1 plays the same role as r of Section IV-A. Analogous to Proposition 4, we obtain the following result for the case of unknown transmit power.

Proposition 6: For $|r_1| \leq 0.993\beta_1$, the interference probability conditioned on E_1 is upper bounded as follows:

$$P_{\text{int}}(a, v|E_1 = r_1) \leq Q(b_1 + b_2|r_1|), \quad (26)$$

where b_1 and b_2 are given in (21) with β replaced by β_1 .

Integrating out r_1 in (26), we get

$$P_{\text{int}}(a, v) \leq Q\left(\frac{b_1}{\sqrt{1 + b_2^2 J_1^{-1}}}\right), \quad (27)$$

which leads to an upper bound on the transmit power s_a by requiring the right-hand side of (27) to be less than ε_{int} . As before, for $s_a^* \neq -\infty$ we require $\beta_1 > \hat{\beta}_1^* > 0$, where $\hat{\beta}_1^* \triangleq \min\{\tilde{\beta}_1 : \Pr(|E_1| < \tilde{\beta}_1) \geq \varepsilon\} = \sqrt{J_1^{-1}} \cdot Q^{-1}\left(\frac{1-\varepsilon}{2}\right)$. Finally, using the invariance principle we obtain the following approximation for the MIFTP:

$$\hat{s}_a = i_{\text{max}} + 10\epsilon \log_{10} \beta_1 - c(\sigma_W, \epsilon, \beta_1, \hat{J}_1, \varepsilon_{\text{int}}), \quad (28)$$

if and only if $\beta_1 > \hat{\beta}_1^* > 0$, where $\hat{\beta}_1^* = \sqrt{\hat{J}_1^{-1}} \cdot Q^{-1}\left(\frac{1-\varepsilon}{2}\right)$.

V. NUMERICAL RESULTS

In this section, we present plots of the MIFTP and the approximate MIFTP estimated from SS measurements under a range of parameter settings. We choose our simulation parameters keeping in mind the application to unused digital television broadcast bands operating in the UHF band [34]³. We consider two cases: i) the transmit power, s_p , of the primary node, p , is known and the FAR nodes only estimate the location \mathbf{L} , and ii) s_p is unknown and the FAR nodes estimate s_p along with \mathbf{L} . The crucial parameters affecting the MIFTP estimation are $d_{p,a}$, s_p , ε_{int} , σ_W , ϵ , N and the CRB $J_{p,a}^{-1}$. We shall assume that the remaining parameters are known constants. Each of the MIFTP values is calculated as an average over 1000 simulation trials and is shown with the associated 95% confidence interval. We place the primary transmitter at location $\mathbf{L} = (50, 50)$ [km] and set the other relevant parameters as follows: $r_{\text{min}} = -83$ dBm, $r_a = -121$ dBm, $\sigma_W = 8$ dB; $\varepsilon_{\text{cov}} = 0.05$, $i_{\text{max}} = -100$ dBm; $\varepsilon_{\text{int}} = 0.01$.

A. Distance $d_{p,a}$

We vary $d_{p,a}$ from 20 to 100 km and position the target FAR node at $\mathbf{L}_a = (x_a, y_a)$, where

$$\mathbf{L}_a = \mathbf{L} + \frac{d_{p,a}}{\sqrt{2}}(1, 1). \quad (29)$$

For a given transmit power of the primary transmitter, $s_p = 80$ dBm, we find $d_{\text{det}}(a)$, the *detection distance* of the FAR nodes. It denotes the radius beyond which the FAR nodes cannot detect the primary signal and is given by $d_{\text{det}}(a) = g^{-1}(s_p - r_a + \sigma_W Q^{-1}(1 - \varepsilon_{\text{cov}}))$, where r_a denotes the

³The FCC recently defined the provisions that allow the operation of unlicensed devices in the TV bands, which is among the first released spectrum for opportunistic secondary access [35].

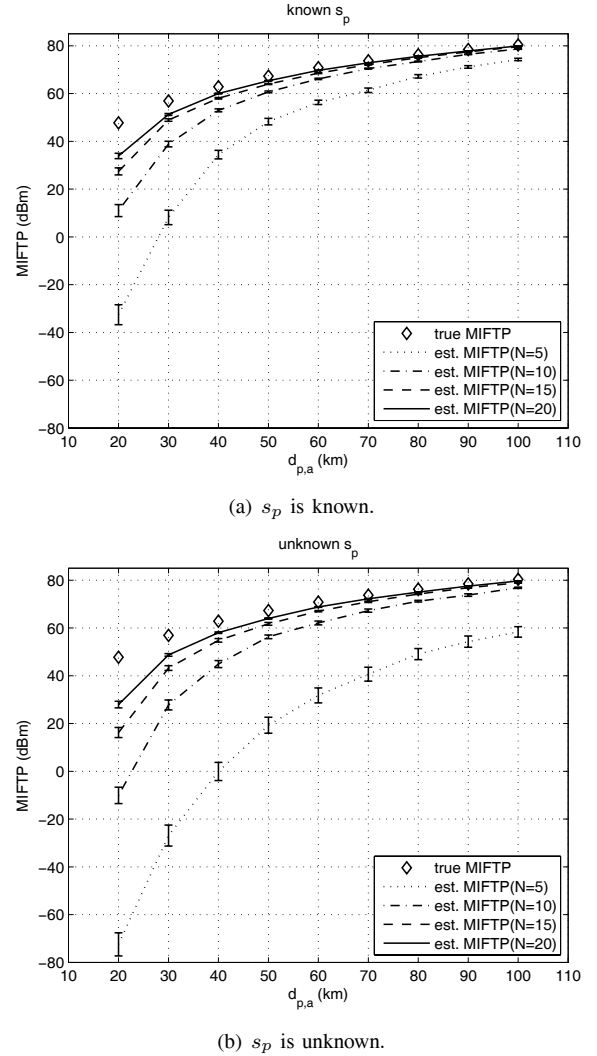


Fig. 4. MIFTP versus $d_{p,a}$, when $\epsilon = 4$.

FAR node's detection threshold. The circular region centered at a with radius $d_{\text{det}}(a)$ is called the *detection region*. For each simulation trial, we randomly place N FAR nodes, with uniform distribution, inside the circle with radius equal to $d_{\text{det}}(a)$ and centered at \mathbf{L} . The set of SS measurements to compute the MLE of \mathbf{L} or Θ is collected by these FAR nodes, which can be used by other far away FAR nodes to estimate the MIFTP. The FAR nodes estimate the MLE of \mathbf{L} assuming a fixed path loss factor $\epsilon = 4$, which is a typical value for the shadowed urban radio propagation. Nodes lying outside the circle with radius $d_{\text{det}}(a)$ use $\hat{\mathbf{L}}_{\text{ML}}$ or $\hat{\Theta}_{\text{ML}}$ to estimate MIFTP based on (25) or (28).

To obtain the ML location and transmit power estimates, the *fmincon* routine of Matlab[®], which employs a sequential quadratic programming method, is used to solve the optimization problems discussed in Section III. As the initial location estimate, we choose the midpoint of the rectangle circumscribing the union of the detection regions of the FAR nodes making the SS measurements. The initial power estimate is set to 60 dBm. Alternatively, one could use the suboptimal estimates derived in [36] as the initial starting point of the optimization problem.

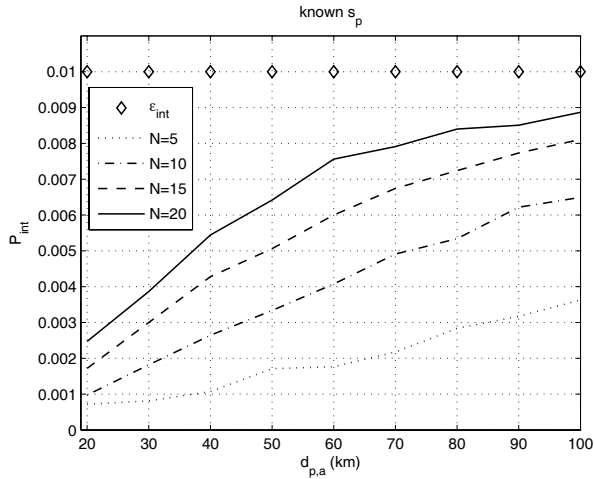


Fig. 5. \hat{P}_{int} versus $d_{p,a}$, when $\epsilon = 4$ and s_p is known.

Figs. 4(a) and 4(b) plot the true and estimated MIFTP values vs. $d_{p,a}$ for both known and unknown s_p when $N = 5, 10, 15, 20$. The confidence intervals shown in the plots arise due to randomness in the localizing FAR node positions, as well as the shadowing noise. We see that the accuracy of the approximate MIFTP formula improves with increasing $d_{p,a}$ and increasing N . In our simulations we found that the MIFTP values depend strongly on the path loss factor ϵ . For larger values of ϵ , the accuracy of the MIFTP approximation improves significantly and the effect of N decreases. This is because, although the received signal becomes weaker as ϵ increases, the sensitivity of the MIFTP approximation on the location estimation error reduces. When $\epsilon = 4$, roughly for $N \geq 10$, the performance degradation due to the estimation of s_p becomes negligible.

We can also calculate the probability of interference, \hat{P}_{int} , which results when the FAR node transmits with power level equal to the MIFTP estimate. Let \hat{s}_a^i denote the MIFTP estimate for the i th simulation trial, $i = 1, \dots, M$. Then the probability of interference under the MIFTP approximation is given by $\hat{P}_{\text{int}} = \frac{1}{M} \sum_{i=1}^M P_{\text{int}}(a|\hat{s}_a^i)$, where $P_{\text{int}}(a|\hat{s}_a^i)$ denotes the interference probability given that the FAR node i transmits with power \hat{s}_a^i (cf. (9)).

Fig. 5 shows the plot of \hat{P}_{int} versus $d_{p,a}$ for $\epsilon = 4$ when s_p is known. We observe that \hat{P}_{int} increases with increasing $d_{p,a}$, but it is always less than ϵ_{int} . When s_p is unknown, \hat{P}_{int} decreases further, since the MIFTP estimate becomes more conservative. For $\epsilon = 5$ (see [32]), \hat{P}_{int} increases as the MIFTP approximation becomes tighter, but always remains smaller than ϵ_{int} . Therefore, the approximate MIFTP can safely be used as an upper bound on the allowable transmit power.

B. Interference probability threshold, ϵ_{int}

In this scenario, we set $\epsilon = 4$, and $d_{p,a} = 50$ km. The location of the FAR node is set according to (29) and the values of the other parameters are set as in the previous scenario. When s_p is unknown, Fig. 6 shows a plot of MIFTP vs. the interference probability threshold, ϵ_{int} , which is varied from 0.001 to 0.1. We see that for $N \geq 10$, the MIFTP increases relatively slowly as ϵ_{int} increases. In particular, the

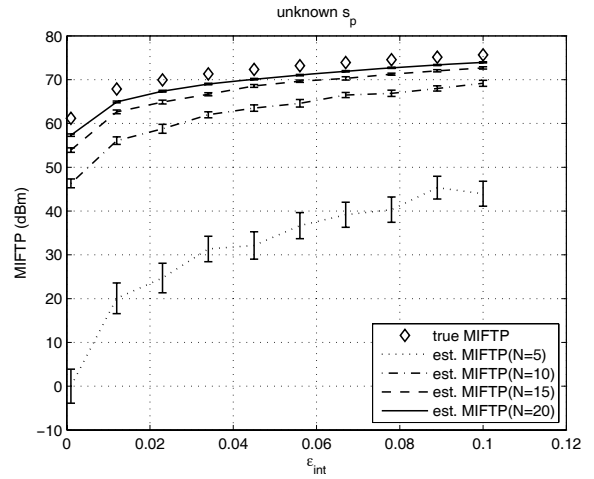


Fig. 6. MIFTP versus ϵ_{int} , when $\epsilon = 4$ and s_p is unknown.

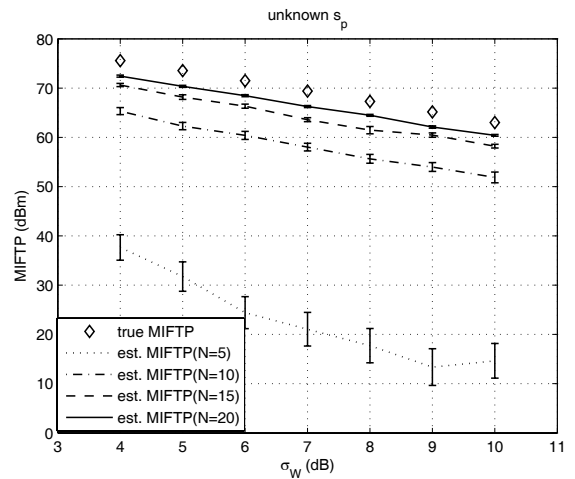


Fig. 7. MIFTP versus σ_W , when $\epsilon = 4$ and s_p is unknown.

difference between the MIFTP value when $\epsilon_{\text{int}} = 0.001$ and when $\epsilon_{\text{int}} = 0.1$ is about 15 dB. Also, the gap between the true and approximate MIFTP values decreases slowly with increasing ϵ_{int} . For $N \geq 10$, the performance degradation due to estimating the unknown s_p is negligible. We have observed that \hat{P}_{int} increases almost linearly with increasing ϵ_{int} , but is always less than the specified threshold.

C. Shadowing noise, σ_W and primary transmit power, s_p

Here, we set $\epsilon_{\text{int}} = 0.01$, keep all other parameters as before, and vary σ_W from 4 to 10 dB. From Fig. 7, we see that the MIFTP decreases almost linearly with increasing shadowing noise variance. The gap between the true and approximate MIFTP values does not depend strongly on the shadowing noise. Again, for $N \geq 10$, the performance degradation due to estimating the unknown s_p is negligible. We have also observed that \hat{P}_{int} does not change appreciably with σ_W and is always less than ϵ_{int} .

Next we set $\sigma_W = 8$ dB, and keep all other parameter values as before. Fig. 8 plots the true and approximate MIFTP values as s_p is varied from 20 to 80 dBm, for unknown s_p . We observe that the accuracy of the approximate MIFTP formula

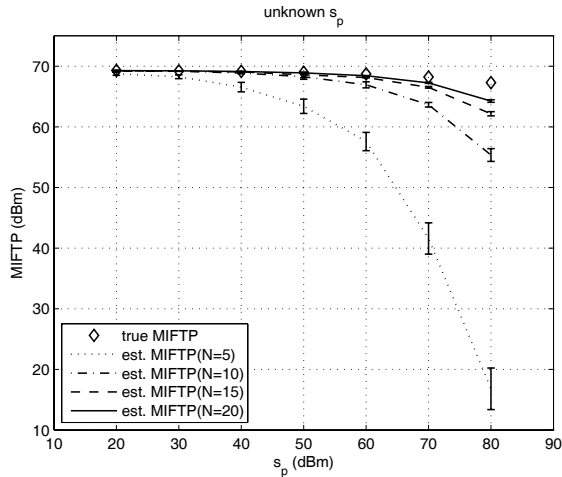


Fig. 8. MIFTP versus s_p , when $\epsilon = 4$ and s_p is unknown.

falls off quickly with increasing s_p when $N \leq 5$. However, increasing N results in a significant improvement in MIFTP accuracy for higher values of s_p . We have also observed that \hat{P}_{int} decreases with increasing s_p as the MIFTP approximation becomes looser and is always less than ϵ_{int} .

VI. CONCLUSION

We developed a collaborative scheme for secondary nodes to compute the approximate maximum interference-free transmit power (MIFTP) in the presence of a primary transmitter with unknown location and transmit power. Knowledge of the MIFTP is necessary for the efficient operation of opportunistic spectrum access methods such as Listen-Before-Talk (LBT) [16] and power control schemes for secondary users [19]. Our numerical results show that the MIFTP is conservative when the number of measurements is small, but becomes more accurate as this number is increased. The property of being conservative is important, as secondary users should “do no harm” to primary users.

Although we have focused on the case of a single primary transmitter, the approach proposed in this paper can be generalized to the case of multiple primary transmitters and FAR nodes simultaneously transmitting over the same frequency channel. The multiple co-channel transmitter case will be addressed in a forthcoming paper. In ongoing work, we are also exploring how other types of information gathered by collaborative FAR nodes can be used to further improve the efficiency of spectrum hole estimation.

APPENDIX

A. Proof of Proposition 2

Using (18), we obtain $\frac{\partial}{\partial \Theta} \ln f_{S|\Theta}(\mathbf{S}) = \frac{1}{\sigma_W^2} \mathbf{BGDW}$, where \mathbf{D} is given by (15), and $\mathbf{B} \triangleq \text{diag} \left[\frac{-10\epsilon}{\ln 10}, \frac{-10\epsilon}{\ln 10}, 1 \right]$ and

$$\mathbf{G} \triangleq \begin{bmatrix} \cos \phi_1 & \cdots & \cos \phi_N \\ \sin \phi_1 & \cdots & \sin \phi_N \\ d_1 & \cdots & d_N \end{bmatrix}. \quad (30)$$

The FIM is given by $\mathbf{J}_{\Theta} = \frac{1}{\sigma_W^2} \mathbf{BGD}^2 \mathbf{G}^T \mathbf{B}$. Using the matrix inversion formula, we obtain (16). It can be shown

that for sufficiently small σ_W^2 , an unbiased estimate exists and the estimation error vector is a linear function of the score function $\frac{\partial}{\partial \Theta} \ln f_{S|\Theta}(\mathbf{S})$ [32]. Therefore, a well-known result in estimation theory [37] allows us to conclude that $\hat{\Theta}_{\text{ML}}$ achieves the CRB asymptotically as $\sigma_W^2 \rightarrow 0$.

B. Proof of Proposition 3

Let $\theta \triangleq d_{p,a}$ denote the true distance between the primary transmitter and the FAR node. The observation equation for estimating θ can be modeled as: $\hat{\mathbf{L}} = \mathbf{u} + \mathbf{W}_L$, where $\mathbf{u} = [u_1, u_2]^T$ with $u_1 = x_a + \theta \cos \phi_{p,a}$ and $u_2 = y_a + \theta \sin \phi_{p,a}$, and $\mathbf{W}_L \sim \mathcal{N}(\mathbf{0}, \mathbf{J}_L^{-1})$. The desired result can be obtained using a similar approach as outlined in Appendix A (see [32] for details).

C. Proof of Proposition 4

We prove the proposition for the case $r \geq 0$. The case $r < 0$ can be proved similarly.

$$P_{\text{int}}(a, v | E_{p,a} = r) = \Pr \{I_v \geq i_{\text{max}} | E_{p,a} = r\} \quad (31)$$

$$\leq \Pr \{W \geq i_{\text{max}} + g(d_a^*) - s_a | E_{p,a} = r\} \quad (32)$$

$$= Q \left(\frac{i_{\text{max}} + \frac{10\epsilon}{\ln 10} \ln(\beta - r) - s_a}{\sigma_W} \right). \quad (33)$$

We expand $\ln(\beta - r)$ in a Taylor series and lower bound it as follows:

$$\ln(\beta - r) = \ln \beta - \sum_{i=1}^{\infty} \frac{1}{i} \left(\frac{r}{\beta} \right)^i \geq \ln \beta - \frac{kr}{\beta},$$

for $0 \leq \frac{r}{\beta} \leq t_k < 1$, where t_k denotes the root of the function $f(t) = \ln(1 - t) + kt$ near 1 with $t \in [0, 1)$ and $k > 0$. The value of t_k can be chosen arbitrarily close to one, for example, when $k = 5$, we have $t_k = 0.993$. Hence,

$$P_{\text{int}}(a, v | E_{p,a} = r) \leq Q(b_1 + b_2 r).$$

REFERENCES

- [1] B. L. Mark and A. O. Nasif, “Estimation of interference-free transmit power for opportunistic spectrum access,” in *Proc. IEEE Wireless Comm. Networking Conf. (WCNC’08)*, Apr. 2008.
- [2] M. McHenry, “Frequency agile spectrum access technologies,” in *Proc. FCC Workshop Cognitive Radio*, May 2003.
- [3] G. Staple and K. Werbach, “The end of spectrum scarcity,” *IEEE Spectrum*, vol. 41, pp. 48-52, Mar. 2004.
- [4] S. Haykin, “Cognitive radio: brain-empowered wireless communications,” *IEEE J. Select. Areas Commun.*, vol. 23, pp. 201-220, Feb. 2005.
- [5] J. Mitola and G. Q. Maguire, “Cognitive radio: making software radios more personal,” *IEEE Personal Commun. Mag.*, vol. 6, pp. 13-18, Aug. 1999.
- [6] J. Mitola, *Cognitive Radio: An Integrated Agent Architecture for Software Defined Radio*. Ph.D. thesis, Royal Institute of Technology (KTH), Stockholm, Sweden, May 2000.
- [7] *Proc. IEEE Symp. on Dynamic Spectrum Access Networks (DySPAN): 2005*, 2007, 2008.
- [8] R. Menon, R. M. Buehrer, and J. H. Reed, “Outage probability based comparison of underlay and overlay spectrum sharing techniques,” in *Proc. IEEE DySPAN*, pp. 101-109, Nov. 2005.
- [9] N. Hoven and A. Sahai, “Power scaling for cognitive radio,” in *Proc. WirelessCom’05*, pp. 250-255, June 2005.
- [10] A. Sahai, N. Hoven, S. M. Mishra, and R. Tandra, “Fundamental tradeoffs in robust spectrum sensing for opportunistic frequency reuse,” tech. rep., University of California, Berkeley, Mar. 2006.
- [11] H. Celebi and H. Arslan, “Utilization of location information in cognitive wireless networks,” *IEEE Wireless Commun. Mag.*, vol. 14, pp. 6-13, Aug. 2007.

- [12] L.-C. Wang and A. Chen, "Effects of location awareness on concurrent transmissions for cognitive ad hoc networks overlaying infrastructure-based systems," *IEEE Trans. Mobile Comput.*, 2009 (to appear).
- [13] S. M. Mishra, R. Tandra, and A. Sahai, "Coexistence with primary users of different scales," in *Proc. IEEE DySPAN*, pp. 158-167, Apr. 2007.
- [14] J. Riihijärvi and P. Mähönen, "Exploiting spatial statistics of primary and secondary users towards improved cognitive radio networks," in *Proc. CrownCom*, May 2008.
- [15] P. Mähönen and M. Petrova and J. Riihijärvi, "Applications of topology information for cognitive radios and networks," in *Proc. IEEE DySPAN*, Apr. 2007.
- [16] A. E. Leu, M. McHenry, and B. L. Mark, "Modeling and analysis of interference in Listen-Before-Talk spectrum access schemes," *Int. J. Network Mgmt.*, vol. 16, pp. 131-147, 2006.
- [17] S. Tang and B. L. Mark, "Performance analysis of a wireless network with opportunistic spectrum sharing," in *Proc. IEEE Globecom*, Washington DC, Nov. 2007.
- [18] Q. Zhao and A. Swami, "A decision-theoretic framework for opportunistic spectrum access," *IEEE Commun. Mag.*, Special Issue on Cognitive Wireless Networks, vol. 14, pp. 14-20, Aug. 2007.
- [19] L. Qian, X. Li, J. Attia, and Z. Gajic, "Power control for cognitive radio ad hoc networks," in *Proc. 15th IEEE Workshop LANMAN*, NJ, U.S.A., pp. 7-12, June 2007.
- [20] K. Hamdi, W. Zhang, and K. B. Letaief, "Power control in cognitive radio systems based on spectrum sensing side information," in *Proc. IEEE Int. Conf. Comm. (ICC'07)*, pp. 5161-5165, June 2007.
- [21] M. Gudmundson, "Correlation model for shadow fading in mobile radio systems," *Electron. Lett.*, vol. 47, pp. 2145-2146, Nov. 1991.
- [22] M. N. Lustgarten and J. A. Madsen, "An empirical propagation model (EPM-73)," *IEEE Trans. Electromagn. Compat.*, vol. 19, Aug. 1977.
- [23] A. G. Longley and P. L. Rice, "Prediction of tropospheric transmission loss over irregular terrain, a computer method - 1968," ESSA 79-ITSS 67, U.S. Dept. of Commerce, Office of Telecommunications, Boulder, CO, July 1968.
- [24] N. S. Adawi *et al.*, "Coverage prediction for mobile radio systems operating in the 800/900 MHz frequency range," *IEEE Trans. Veh. Technol.*, vol. 37, pp. 3-44, 1988.
- [25] Y. Qi, H. Kobayashi, and H. Suda, "Analysis of wireless geolocation in a non-line-of-sight environment," *IEEE Trans. Wireless Commun.*, vol. 5, pp. 672-681, Mar. 2006.
- [26] S. Capkun, M. Hamdi, and J.-P. Hubaux, "GPS-Free positioning in mobile ad-hoc networks," in *Proc. 34th Hawaii Int. Conf. System Sciences*, Jan. 2001.
- [27] D. Moore, J. Leonard, D. Rus, and S. Teller, "Robust distributed network localization with noisy range measurements," in *Proc. 2nd ACM SenSys*, Nov. 2004.
- [28] P. Biswas and Y. Ye, "Semidefinite programming for ad hoc wireless sensor network localization," in *Proc. 3rd Int. Symp. Inform. Processing Sensor Networks, IPSN 2004*, pp. 46-54, Apr. 2004.
- [29] L. Doherty, K. S. J. Pister, and L. E. Ghaoui, "Convex position estimation in wireless sensor networks," in *Proc. IEEE INFOCOM'01*, vol. 3, pp. 1655-1663, Apr. 2001.
- [30] Y. Qi, "Wireless geolocation in a non-line-of-sight environment," Ph.D. thesis, Princeton University, Princeton, NJ, 2003.
- [31] L. L. Scharf, *Statistical Signal Processing: Detection, Estimation, and Time Series Analysis*. Addison-Wesley, 1991.
- [32] B. L. Mark and A. O. Nasif, "Estimation of maximum interference-free power level for opportunistic spectrum access," Tech. Rep. GMU-NAPL-Y08-N1, George Mason University, NAPL, Aug. 2008. [Online] Available: <http://napl.gmu.edu>.
- [33] S. Verdú, *Multuser Detection*. New York: Cambridge University Press, 1998.
- [34] "Longley-Rice Methodology for evaluating TV coverage and Interference," Tech. Rep. OET Bulletin 69, Office of Engineering and Technology (OET), Federal Communications Commission, Feb. 2004.
- [35] C. Cordeiro, K. Challapali, and D. Birru, "IEEE 802.22: an introduction to the first wireless standard based on cognitive radios," *IEEE J. Commun.*, vol. 1, pp. 38-47, Apr. 2006.
- [36] S. Kim, H. Jeon, and J. Ma, "Robust localization with unknown transmission power for cognitive radio," in *Proc. IEEE Int. Conf. Military Commun. (Milcom'07)*, Oct. 2007.
- [37] H. L. Van Trees, *Detection, Estimation and Modulation Theory: Part I*. New York: John Wiley & Sons, Inc., 2001.



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